Physical Interpretation of the Solution to the Problem of Diffraction on a Half-plane with Non-Ideal Boundary Conditions

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ABSTRACT: The recently proposed method of fundamental components is employed to develop a technique for obtaining a heuristic solution to the problem of diffraction on a half-plane with non-ideal boundary conditions. The difference between the new method and traditional heuristic approaches, such as the geometric theory of diffraction and the method of edge waves, is the presence of an adjustment procedure which allows increasing the accuracy while maintaining the compactness of the formulas. For the case of the problem of diffraction on an impedance half-plane, heuristic formulas are constructed. Then they are refined using a verification solution. A quantification of accuracy is carried out, and a physical interpretation of the solution is presented. The prospects for applying this approach to constructing high-speed solvers and carrying out the physical interpretation of numerical solutions are discussed.

KEYWORDS: Electromagnetic Diffraction, Heuristic Approaches, Impedance Boundary Conditions

1. Introduction

Topical practical problems, such as the study of scattering on targets with low radar visibility, the propagation of electromagnetic waves in urban areas, etc., require the use of special solvers. These solvers, in turn, require efficient (i.e., fast and accurate) methods for solving diffraction problems [1].

In the topical problems mentioned above, which can be classified as diffraction by polygons and polyhedra, there are many edges with various shapes and boundary conditions. Computer resources may not be enough to apply numerical methods, and it is unlikely to obtain rigorous analytical solutions for all of them. One has to apply heuristic formulas [2 – 5]. The recently developed Method of Fundamental Components (MFC) [6 – 8] offers an efficient solution to this problem. Heuristic formulas obtained employing MFC are compact, fast, and, unlike known heuristic approaches, accurate in an entire range of parameters [8].

The MFC heuristic solution is based on a set of fundamental components [8]. At the first stage it is the solution of the simplest problems of diffraction. Further, these solutions are refined using other basic components and a verification solution (usually, numerical or numerical-analytical one).

The MFC application in order to obtain heuristic formulas describing the solution to the problem of diffraction on a semitransparent half-plane is based on the following algorithm [6, 8, 9]:

- Development of a verification solution.
- Selection of primary heuristic formulas from the number of known solutions.
- Carrying out the adjustment procedure in order to refine the primary heuristic formulas by comparing them with the verification solution.

Further, when constructing the heuristic solution of the MFC, we will follow this sequence of actions.

One of the fundamental components is the singular diffraction coefficient for a semi-infinite scatterer [6, 8, 9]. Obtaining heuristic formulas for two-dimensional semi-infinite scatterers, even in spite of the existence of rigorous solutions, is a topical problem [9]. In this study we will analyze heuristic formulas for the problem of diffraction on an impedance half-plane and give a physical
interpretation of the verification by means of the Wiener-Hopf method (WHM) solution.

The present study is devoted to the technique of constructing heuristic solutions to diffraction problems on two-dimensional semi-infinite objects with non-ideal boundary conditions. Such solutions can be further applied to construct heuristic solutions of diffraction problems on three-dimensional objects. To build heuristic solutions, one can use any reliable solution, mostly numerical. The Wiener-Hopf method satisfies all the required conditions.

The previous works of the author were devoted to obtaining individual specific solutions. This article systematizes previous results, formulates a general methodology and calculates quantification of accuracy.

Let us consider two types of wave excitation: TH-polarization or TE- polarization (magnetic field vector $\vec{H}$ or electrical field vector $\vec{E}$ is perpendicular to the edge, correspondingly).

2. Construction of a heuristic solution for TH-polarization

2.1. Development of a verification solution

Consider the problem of electromagnetic wave diffraction on an impedance half-plane. The boundary conditions for the case of TH-polarization (vector $\vec{H}$ is perpendicular to the edge) have the form [9 – 16]:

$$
\begin{align*}
H_{x+} - H_{x-} &= -Z^{-1}E_z, \\
E_{z+} &= E_{z-} = E_z \text{ when } y = 0,
\end{align*}
$$

(1)

where impedance $Z=iX$, $X$ is a variable parameter, which the reflection coefficient depends on, and $i$ is an imaginary unit. Here the signs “+” and “-” correspond to the regions $y>0$ and $y<0$, respectively (see Figure 1).

![Figure 1: The geometry of the problem.](image)

The reflection and transmission coefficients $R_{TH}$ and $T_{TH}$ for an unbounded half-plane with boundary conditions (1) on the surface can be written as follows:

$$R_{TH}(X, \varphi_0) = \frac{-W_0}{W_0 + 2i\sin(\varphi_0)X}, \quad W_0 = 120\pi .$$

(2)

$$T_{TH} = 1 + R_{TH} .$$

(3)

The verification solution:

$$fr(X, \varphi, \varphi_0)$$

(4)

is obtained employing the Wiener-Hopf method [9 – 13].

2.2. Selection of primary heuristic formulas

We choose primary heuristic formulas as in [8]:

$$fg(R, T, \varphi, \varphi_0) = \frac{1}{2} \left( \frac{1 - T}{-\cos \frac{\varphi - \varphi_0}{2}} + \frac{R}{-\cos \frac{\varphi + \varphi_0}{2}} \right).$$

(5)

$$fg^0(R, T, \varphi, \varphi_0) = \frac{1}{2} \left( \frac{1 - T}{\sin \frac{\varphi - \varphi_0}{2}} + \frac{R}{\sin \frac{\varphi + \varphi_0}{2}} \right).$$

(6)

In (5) and (6): $R=R_{TH}$, $T=T_{TH}$. When $X=0$: $R_{TH}=-1$, $T_{TH}=0$, and expressions (5), (6) are transformed into known expressions $f$, $f^0$ from [17 – 19]. Formula (5) corresponds to the generalized diffraction coefficients (GDC) approximation, and formula (6) corresponds to the physical optics diffraction coefficients (PODC) approximation [8].

2.3. Carrying out the adjustment procedure

The adjustment procedure consists in comparing the heuristic formula (5) with the Wiener-Hopf solution (4) [9]. At the first stage, we compare the functions (4) and (5) and empirically find the semitransparency function depending on $(X, \varphi)$ [10, 11], which allows us to bring the primary formula closer to a rigorous solution:

$$cxf(X, \varphi) = 1 - x(X) \left[ 1 - \cos \left( \frac{\pi - \varphi}{1 + x(X)} \right) \right],$$

(7)

$$x(X) = 1 - \exp \left( -\frac{X}{W_0} \right),$$

(8)

$$fh(R, T, \varphi, \varphi_0) = fg(R, T, \varphi, \varphi_0) \frac{cxf(X, \varphi_0)}{cxf(X, \varphi)} .$$

(9)

Here, $cxf(X, \varphi)$ is the function $cxf(X, \varphi)$ value at the singularity point angle $\varphi_0 = \pi - \varphi_0$, that is disposed on the boundary between light and shadow.

3. Construction of a heuristic solution for TE-polarization

3.1. Development of a verification solution

The boundary conditions for the case of the incidence of a TE-polarized wave (vector $\vec{E}$ is perpendicular to the edge) on an impedance half-plane have the form [9 – 16]:

$$T_{TH} = 1 + R_{TH} .$$

(3)
\[
\begin{align*}
Z(H_{z^+} - H_{z^-}) &= E_x, \\
E_{x^+} &= E_{x^-} = E_x \text{ when } y = 0. 
\end{align*}
\] (10)

The geometry of the problem is depicted in Figure 2.

![Figure 2: The geometry of the problem](image)

The reflection and transmission coefficients \( R_{TE} \) and \( T_{TE} \) for an unbounded half-plane with boundary conditions (10) on the surface can be written as follows:

\[
R_{TE}(X, \varphi_0) = \frac{W_0}{W_0 + 2W_0 \sin(\varphi_0)} - 1, \quad W_0 = 120\pi. (11)
\]

\[
T_{TE} = 1 - R_{TE}. (12)
\]

The verification solution:

\[
gr(X, \varphi, \varphi_0). (13)
\]

is obtained employing the Wiener-Hopf method [9 – 13].

3.2. Selection of primary heuristic formulas

Primary heuristic formulas we choose as in [8]:

\[
\begin{align*}
\text{gf}(R, T, \varphi, \varphi_0) &= \frac{1}{2} \left( \frac{1 - T}{-\cos \frac{s - \varphi_0}{2}} + \frac{R}{-\cos \frac{\varphi - \varphi_0}{2}} \right), \quad (14) \\
\text{gf}^0(R, T, \varphi, \varphi_0) &= \frac{1}{2} \left( \frac{(1 - T)\sin \frac{s - \varphi_0}{2} + R\sin \frac{\varphi - \varphi_0}{2}}{-\cos \frac{s - \varphi_0}{2}} \right), \quad (15)
\end{align*}
\]

In (14) and (15): \( R = R_{TE}, \quad T = T_{TE}. \) When \( X = 0; R_{TH} = 1, T_{TH} = 0, \) and expressions (14), (15) are transformed into known expressions \( g, g^0 \) from [17 – 19]. The difference between formulas (14), (15) and formulas (5), (6) is that we substitute different values of \( R \) and \( T \) into each one. Formula (14) corresponds to the GDC approximation, and formula (15) corresponds to the PODC approximation [8].

3.3. Carrying out the adjustment procedure

Just as for the case of the TH polarization described in sub-section 2.3, we empirically select the semitransparency function depending on \((X, \varphi)\) [10, 11] for the TE- polarization:

\[
cxg(X, \varphi) = \left[ \sin(\varphi/2) \right]^{(X)}. (16)
\]

\[
x(X) = 1 - \exp(-X/W_0).
\]

Heuristic formula for TE- polarization:

\[
gh(R, T, \varphi, \varphi_0) = \text{gf}(R, T, \varphi, \varphi_0)\frac{cxg(X, \varphi)}{cxg(X, \varphi_0)}. (17)
\]

Here also \( cxg(X, \varphi) \) is the function \( cxg(X, \varphi) \) value at the singularity point angle \( \varphi_0 = \pi - \varphi_0 \), that is disposed on the boundary between light and shadow.

4. Calculation results

Figure 3 depicts the calculation results for TH-polarization at \( \varphi = 90^\circ \) and different values of \( X \).

A study of the behavior of the functions whose graphs are depicted in Figure 3 showed that as the parameter \( X \) changes from 0 to infinity, the shapes of curves GDC (5) and PODC (6) remain unchanged, while curves (4) and (9) shift from the curve (5) to curve (6).

![Figure 3 (a) X=50 x(X)=0.124](image)

![Figure 3 (b) X=300 x(X)=0.549](image)
The angle $\varphi$ is depicted horizontally. The scattering diagrams (SD) modules are plotted vertically. The solid gray line is WHM solution $fr(X, \varphi, \psi)$ (4). circles are the GDC formula (5), triangles are the PODC formula (6), squares are the heuristic solution $fr(R, T, \varphi, \psi)$ (9).

A study of the behavior of the functions whose graphs are depicted in Figure 4 showed that as the parameter $X$ changes from 0 to infinity, the shapes of curves GDC (14) and PODC (15) remain unchanged, while curves (13) and (17) shift from the curve (14) to curve (15).

Figure 4 illustrates the calculation results for TE-polarization at $X=100$, $\varphi=90^\circ$.

5. Quantification of accuracy

Quantification of accuracy was carried out employing formulas, which we will call "functions $L$":

$$Lfh(R,T,\varphi,\psi_0) = \log \left( \frac{1}{2\pi} \int_{\psi_0}^{2\pi} \frac{fr(X, \varphi, \psi_0)}{fh(R, T, \varphi, \psi_0)} \, d\psi \right),$$

(18)

$$Lfg(R,T,\varphi,\psi_0) = \log \left( \frac{1}{2\pi} \int_{\psi_0}^{2\pi} \frac{fr(X, \varphi, \psi_0)}{fg(R, T, \varphi, \psi_0)} \, d\psi \right),$$

(19)

$$Lfg^0(R,T,\varphi,\psi_0) = \log \left( \frac{1}{2\pi} \int_{\psi_0}^{2\pi} \frac{fr(X, \varphi, \psi_0)}{fg^0(R, T, \varphi, \psi_0)} \, d\psi \right),$$

(20)

$$Lgh(R,T,\varphi,\psi_0) = \log \left( \frac{1}{2\pi} \int_{\psi_0}^{2\pi} \frac{gr(X, \varphi, \psi_0)}{gh(R, T, \varphi, \psi_0)} \, d\psi \right),$$

(21)

$$Lgf(R,T,\varphi,\psi_0) = \log \left( \frac{1}{2\pi} \int_{\psi_0}^{2\pi} \frac{gr(X, \varphi, \psi_0)}{gf(R, T, \varphi, \psi_0)} \, d\psi \right),$$

(22)

$$Lgf^0(R,T,\varphi,\psi_0) = \log \left( \frac{1}{2\pi} \int_{\psi_0}^{2\pi} \frac{gr(X, \varphi, \psi_0)}{gf^0(R, T, \varphi, \psi_0)} \, d\psi \right).$$

(23)

Each of the functions $L$ is a quantitative integral estimate of the accuracy of the function in the denominator of the integrand on a logarithmic scale. The smaller the value of a function $L$ is, the closer the corresponding function in the denominator of the integrand is to the verification solution.

Graphs quantitatively describing the deviations of heuristic approximations from the rigorous solution for TH-polarization are depicted in Figure 5.
wave on a non-transparent half-plane [11, 20]. Generalized bilateral impedance boundary conditions (GBIBC), for which boundary conditions (1) are a special case, lead to the following expressions for the diffraction coefficient:

$$f_{gh}(R, T, \varphi, \varphi_0) = \frac{1}{2} \left( \frac{1}{1 - \cos \varphi_0} + \frac{1}{1 - 2 \arctan(R) / \pi + C} \right).$$ (24)

Here \( R \) is the reflection coefficient depending on the parameter \( X1 \), similar to \( X \) [11, 20]. \( C \) is a small parameter for additional accuracy correction. In further work in accordance with the MFC concept on the heuristic formula (24), in order to apply it in a specific practical problem, the parameter \( C \) should either be distributed over other components of the heuristic formula (24), or turned into a basic component that has a physical meaning. In accordance with the GBIBC, in (24) for all \( X \) values [20]: \(|R| = 1, T = 0\), i.e. the half-plane is completely non-transparent, and only the phase of the reflection coefficient \( R \) changes. The graphs of the behavior of the function (24) are depicted in Figure 7.

6. Physical interpretation of the solution

As \( X \) changes from 0 to \( \infty \), the following changes occur: the expression \( fh(R, T, \varphi, \varphi_0) \) (9) transforms from the formula \( fg(R, T, \varphi, \varphi_0) \) (5) to the formula \( fg(R, T, \varphi, \varphi_0) \) (6), and the expression \( gh(R, T, \varphi, \varphi_0) \) (17) transforms from the formula \( gf(R, T, \varphi, \varphi_0) \) (14) to the formula \( gf(R, T, \varphi, \varphi_0) \) (15).

This means that the greater the transparency of the object is, the more the formulas for the GDC approximation (5), (14) are transformed into formulas for the PODC approximation (6), (15). This property can be used in the future to build primary heuristic formulas for semi-transparent objects.

Thus, none of the approximations GDC or PODC taken separately gives satisfactory solution in the entire range of values of the parameter \( X \) [8, 9, 11].

7. Non-transparent half-plane

As an additional illustration of the capabilities of the MFC, we present the calculated data for solving the problem of diffraction of a TH-polarized electromagnetic

![Figure 7](image_url)

Figure 7 (a) \( X1=20, \varphi=60, C=0 \)

Figure 7 (b) \( X1=400, \varphi=90, C=0 \)

Figure 7 (c) \( X1=2000, \varphi=60, C=0 \)
Figure 7: The TH-polarization scattering diagrams modules. The angle $\phi$ is plotted horizontally. The scattering diagrams (SD) modules are plotted vertically. The solid gray line is WHM solution [11, 20], the squares are the heuristic solution $g_f(H(R, T, \phi, \phi_0))$ (24).

7.1. Physical interpretation of the heuristic solution of the problem of diffraction on a non-transparent half-plane

For TH-polarization, as a result of the change in the phase of the reflection coefficient $R$ as the parameter similar to $X$ changes from 0 to $\infty$, the transformation takes place: $f \rightarrow g$ [11, 20].

8. Discussion

In this article, heuristic formulas are obtained for solving problems of diffraction on a semitransparent half-plane. A refinement over the GDC and PODC approximations was found employing the method of fundamental components (MFC). Heuristic formulas similar to GDC have been proposed many years ago [21, 22], although they were applied without an adjustment procedure.

Obtaining refined heuristic solutions for two-dimensional scatterers with non-ideal boundary conditions is a topical problem [23 – 26]. Such solutions are obtained even in the presence of reliable numerical solutions that can be used for verification of the mentioned heuristic formulas. The purpose of obtaining refined heuristic solutions is to use them in solvers designed for resource-intensive practical problems.

Together with heuristic approaches that describe the shape of the contour and edge profile, as well as the influence of the far zone condition [6, 8], the approach that describes the influence of boundary conditions complements the set of fundamental components that allows you to build effective heuristic formulas for three-dimensional objects of complex shape and with various boundary conditions.

In addition to refining the primary heuristic formulas, the MFC can be used to correct the numerical solution based on the experimental results. Such a need may arise in those cases when, due to inaccurate input data, the numerical calculation diverges from the experimental results. Engineering approaches of this kind are already widely used, however, unlike MFC, they are usually not based on primary heuristic formulas which have a physical meaning that helps to understand the reasons for the discrepancies between theory and experiment.

The MFC approach can be used to calculate diffraction for large 3D objects, for which it is impossible to obtain a numerical solution. The procedure for adjusting the heuristic solution is carried out on a smaller size scatterer. Having achieved the required accuracy, one can further apply the heuristic formula for a larger scatterer, which can no longer be calculated by rigorous numerical methods. In this case, the performance of heuristic formulas will not change, and the accuracy will even increase compared with the smaller size scatterer.

The direction of future research is to apply the technique proposed in this article to objects of different shapes and with different boundary conditions. In the future, such solutions can significantly increase the speed of solvers and give them new opportunities for solving practical problems.

9. Conclusions

The article proposes a technique for improving the accuracy of primary heuristic formulas employing the adjustment procedure. The technique has been verified for boundary conditions of certain types. However, this approach can be just as well applied to a half-plane with arbitrary boundary conditions, provided that a verification solution is available.
Conflict of Interest

The author declares no conflict of interest.

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