1. Introduction

Many missile guidance and control laws have been developed in the past. Reported missile autopilot designs have ranged from classical linear to the most advanced ones [1]–[4]. The most widely used guidance law among them is a variant of proportional navigation (PN) [5]–[8]. Homing missiles using a variant of proportional navigation (PN) guidance law require measurement of LOS angular rate and measurement of closing velocity or missile velocity. This implies that missile seeker is able to detect and track targets during engagement. In active homing missiles, radar based missile seeker is used to detect the target, track the target and measure LOS rate and closing velocity. In passive homing missiles, strap-down or gimbaled imaging sensor (EO/IIR) based missile seeker is used to detect the target, track the target and estimate LOS angular rate, while missile velocity is measured with an appropriate sensor (IMU). In this work, a two axis (pitch and yaw) gimbal carrying an imaging device keeps the target in the camera field of view (FOV) by means of a gimbal control tracking loop. Accurate measurement of tracking error is of essential importance for tracking accuracy and LOS rate estimation. Tracking error is obtained by locating targets within the image using appropriate image processing techniques. Thanks to rapid advances in deep machine learning (convolutional neural networks), high accuracy and high speed deep learning object detection models have emerged. The most prominent among them is the YOLO (You only look once) family [9]. This work investigates application of YOLO deep machine learning object detector for tracking error measurements and consecutive LOS angular rate estimation in missile seeker. YOLO performance assessment in guidance loop is realized by means of a comprehensive Matlab/Simulink simulator, which includes nonlinear 6DOF missile and controls dynamics, 3D world and camera model, is developed. Intensive simulation results show performances of the proposed missile homing system.

KEYWORDS Missile guidance, YOLO, Missile seeker, Proportional navigation, 6DOF

2. Missile 6-DOF dynamic model

In order to simulate missile guidance, a high-fidelity nonlinear model is needed. Cruciform missile can be modeled as a rigid body which is governed by a system of coupled nonlinear first order differential equations with constant coefficients:

\[ P = Lu \]  
\[ Q = PR(I_x - I_z)/I_z + M/I_z \]  
\[ R = PQ(I_y - I_z)/I_z + N/I_z \]  
\[ \dot{u} = vR - wQ + F_x/m \]  
\[ \dot{v} = WP - uR + F_y/m \]  
\[ \dot{w} = uQ - vP + F_z/m \]
Where \( u, v \) and \( w \) are missile velocities along \( x, y \) and \( z \) axes of the missile body frame. \( P, Q \) and \( R \) are rotational angular velocities about \( x, y \) and \( z \) axes of the missile body frame. \( F_x, F_y \) and \( F_z \) are forces which are acting along the respected body axes and \( L, M \) and \( N \) are moments which are acting along the respected body axes. \( I_x, I_y \) and \( I_z \) are moments of inertia about the missile body axes. Inertia cross products are zero since the missile airframe is symmetric. For a cruciform missile, aerodynamic forces can be approximated as follows [10]:

\[
\begin{pmatrix}
F_x \\
F_y \\
F_z \\
\end{pmatrix} =
\begin{bmatrix}
C_{s_x} + C_{s_2} (\alpha^2 + \beta^2) \\
C_{s_\theta} + C_{s_\theta} (\alpha^2 + \beta^2) \\
C_{s_\phi} + C_{s_\phi} (\alpha^2 + \beta^2)
\end{bmatrix}
\begin{pmatrix}
\frac{\dot{\phi}}{\cos \phi} \\
\frac{\dot{\theta}}{\cos \theta} \\
\frac{\dot{\psi}}{\cos \psi}
\end{pmatrix}
\]

Total force on acting along the missile body axes can:

\[
\begin{bmatrix}
F_x \\
F_y \\
F_z \\
\end{bmatrix} = \begin{bmatrix} T \\
0 \\
0 \end{bmatrix} + mg \begin{bmatrix}
\sin \theta \\
-\sin \phi \cos \theta \\
\cos \phi \cos \theta
\end{bmatrix} - qS \begin{bmatrix}
C_{s_x} + C_{s_2} (\alpha^2 + \beta^2) \\
C_{s_\theta} + C_{s_\theta} (\alpha^2 + \beta^2) \\
C_{s_\phi} + C_{s_\phi} (\alpha^2 + \beta^2)
\end{bmatrix}
\begin{bmatrix}
\frac{\dot{\phi}}{\cos \phi} \\
\frac{\dot{\theta}}{\cos \theta} \\
\frac{\dot{\psi}}{\cos \psi}
\end{bmatrix}
\]

Where \( g = \rho g^2/2 \) is the dynamic pressure, \( g \) is the density of air at given altitude and \( v_{\text{NA}} \) is the total velocity of the missile. The gravitational force is naturally given in earth reference frame so it has to be transformed to body axes frame. In the previous equation, \( C_{s_x} \) and \( C_{s_\theta} \) are aerodynamic coefficients calculated at the various angles of attack \( \alpha \) and sideslip angles \( \beta \). Angle of attack and sideslip angle can be calculated as follows:

\[
\begin{align*}
\alpha &= \arctan \left( \frac{w}{v} \right) \\
\beta &= \arctan \left( \frac{v}{u} \right)
\end{align*}
\]

Furthermore, it is important to define Euler RPY angles which describe the orientation of the missile with respect to the inertial reference frame. Euler angles can be integrated from the following equation:

\[
\begin{pmatrix}
\phi \\
\theta \\
\psi
\end{pmatrix} =
\begin{bmatrix}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi \csc \theta & \cos \phi \csc \theta
\end{bmatrix}
\begin{pmatrix}
P \\
Q \\
R
\end{pmatrix}
\]

Moments acting on the missile are a consequence of rotation of actuator surfaces and of forces acting on the missile. Aerodynamic forces are acting on the point called the center of pressure, while the gravitational force is acting on the center of gravity of the missile. This causes a moment which rotates the missile. Therefore, the missile moments can be calculated as follows:

\[
\begin{pmatrix}
L \\
M \\
N
\end{pmatrix} =
\begin{bmatrix}
-r_x F_{AZ} \\
r_x F_{AY} \\
r_x F_{AZ}
\end{bmatrix} + qS \begin{pmatrix}
C_{L_\phi} \delta_{\psi} \\
C_{M_\psi} \delta_{\phi} \\
C_{M_\phi} \delta_{\psi}
\end{pmatrix}
\]

In the previous equation, \( r_x \) is the distance between center of gravity and center of pressure and \( F_{AY} \) and \( F_{AZ} \) are aerodynamic forces acting along the missile body axes. \( C_{L_\phi}, C_{M_\psi} \) and \( C_{M_\phi} \) are rolling, pitching and yawing aerodynamic coefficients respectively. \( C_{L_{\phi}}, C_{L_{\psi}} \) and \( C_{L_{\phi}} \) are aerodynamic coefficients with respect to the angular displacement of actuator surfaces denoted by \( \delta_{\psi}, \delta_{\phi} \) and \( \delta_{\phi} \). Table 1 shows used missile parameters [11].

3. Proportional navigation

The basic intuition of proportional navigation (PN) [12] is to generate commanding lateral accelerations which are proportional to the line of sight rate (LOS) angular rate as follows:

\[
\alpha_c = -N \lambda v_c
\]

Where \( \alpha_c \) is the commanded lateral acceleration normal to the line of sight, \( N \geq 2 \) is the effective navigational ratio and \( v_c \) is the relative velocity between the missile and the target. These commanded accelerations are then forwarded to the autopilot which makes sure that the missile achieves these lateral accelerations. In recent works, authors simulated this guidance method on 6-DOF missile simulator in Matlab in [13] and [14]. For even more insight into missile guidance and control, the reader is referred to [10], [11], [15]–[21]. As is shown in figure 1, in order to calculate \( \lambda \) or \( \dot{\lambda} \) it is needed to measure relative distance and relative velocity between the missile and the target.

Unfortunately, these values cannot be measured without an active radar which most tactical missiles do not have. Similarly, the closing velocity \( v_c \) cannot be measured without an active radar. That is why this article implements another variant of PN called the pure proportional navigation whose commanding accelerations are normal to the missile velocity [12] and are given by:

\[
\alpha_c = N \lambda v_m
\]

Table 1: Missile parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_x )</td>
<td>0.024 kgm^2</td>
</tr>
<tr>
<td>( I_y, I_z )</td>
<td>0.958 kgm^2</td>
</tr>
<tr>
<td>( m )</td>
<td>11.25 kg</td>
</tr>
<tr>
<td>( C_{s_x} )</td>
<td>0.484</td>
</tr>
<tr>
<td>( C_{s_y} )</td>
<td>0.595</td>
</tr>
<tr>
<td>( C_{s_\theta} )</td>
<td>0.645</td>
</tr>
<tr>
<td>( C_{s_\phi} )</td>
<td>0.578</td>
</tr>
<tr>
<td>( C_{s_\phi} )</td>
<td>0.595</td>
</tr>
<tr>
<td>( C_{s_\phi} )</td>
<td>0.578</td>
</tr>
<tr>
<td>( C_{s_\phi} )</td>
<td>0.595</td>
</tr>
<tr>
<td>( C_{s_\phi} )</td>
<td>0.578</td>
</tr>
</tbody>
</table>

Missile seeker is the device that has to measure LOS rate. The seeker is made out of two independent gimbal axes, two...
servo motors and an imaging (EO/IIR) sensor. These two gimbal motors move across the vertical and the horizontal plane in order to lock onto the target. Once locked, the seeker can measure the tracking error, which is noted with $\epsilon$ in figure 1. Earlier work [14], [22]–[25] explain in great detail on how to obtain LOS and LOS rate from the measured tracking error. The LOS angular rate in vertical plane can be obtained using the following equation:

$$\dot{\lambda}_{vz} = \epsilon_y \frac{s}{Ts + 1} + \dot{\psi}_h + \dot{\psi}_m$$  \hspace{1cm} (15)

Where $\dot{\psi}_m$ is the Euler pitch angle of the missile relative to the inertial frame of reference, $\dot{\psi}_h$ is the angular position of the gimbal servo relative to the missile centerline in vertical plane and $\epsilon_y$ is the measured tracking error in the vertical plane. Angular velocity $\dot{\psi}_h$ is easy to obtain using rate gyro as is explained in [14] and $\dot{\psi}_h$ has to be calculated from measured angular rates around the body axes as is explained in [13]. Similar to equation 15, LOS rate in the horizontal plane can be obtained as follows:

$$\dot{\lambda}_{vy} = \epsilon_x \frac{s}{Ts + 1} + \psi_h + \psi_m$$  \hspace{1cm} (16)

Where $\psi_m$ is the Euler yaw angle of the missile relative to the inertial frame of reference, $\psi_h$ is the angular position of the gimbal servo relative to the missile centerline in the horizontal plane and $\epsilon_x$ is the measured tracking error in the horizontal plane. Most tactical missiles have infrared homing seeker. Because of this, tracking error cannot be measured directly but it is assumed to be indicated angular position of the target relative to the camera centerline or boresight [10]. This paper aims to simulate a seeker sensor which utilizes Matlab 3D camera, which would measure the tracking error from obtained images. Matlab interface for Unreal Engine allows simulations of 3D vehicles as well as taking images from Unreal Engine world. Also, implementation, simulation and control of gimbal servo system are presented in this article since it is guidance requirement to keep the tracking error as low as possible.

4. Autopilot design

If the missile lateral accelerations are exactly equal to the commanded accelerations commanded by the guidance subsystem, then the interception is guaranteed. Missile subsystem which will ensure this is referred to as the autopilot. From the missile dynamic model, it can be shown that there will be no cross-coupling between yaw and pitch motion if the missile is not rolling. Therefore, it is imperative that the autopilot ensures zero roll angle of the missile. Once that is achieved, motion in yaw and pitch plane can be controlled independently. Therefore, it is enough to implement three PID controllers. One PID controller to ensure roll stability and two PID controllers for two lateral accelerations. Figure 2 shows a roll controller design.

Roll controller employs a cascade error regulation. The inner loop which feeds back roll derivative is a damping loop which ensures minimum oscillations in the roll channel. In order to achieve lateral acceleration tracking, an expression for calculating lateral acceleration is needed. In order to calculate them, a derivative of a rotating vector is needed and that vector has to be transformed into coordinate system whose $x$-axis coincides with the velocity vector. Such coordinate system is also called the wind frame. Therefore, accelerations in the wind frame are given by:

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ \sin \beta \cos \alpha + \cos \beta \sin \alpha \\ -\sin \alpha \cos \beta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \end{bmatrix}$$

\hspace{1cm} (17)

Where $a_x$ and $a_z$ are missile horizontal and vertical acceleration components normal to the velocity vector and $a_y$ is the missile acceleration component along the velocity vector. This acceleration is usually zero since the missile does not have any fuel left during the terminal guidance phase. Figure 3 shows a controller for lateral accelerations. Similar design for horizontal and vertical acceleration components is employed. The inner loop reduces response oscillations and the outer loop ensures that the lateral acceleration follows the demanded lateral acceleration.

4. YOLO neural network

In order to calculate the tracking errors $\epsilon_x$ and $\epsilon_y$, it is first required to localize object in the image. Furthermore, it is needed to classify the object in the obtained image. There is a handful of requirements that the localization algorithm
has to satisfy in missile guidance applications. First, during simulations images are obtained from Unreal Engine environment so the algorithm must be able to work with artificial images. Second, the algorithm has to be very fast since camera can generate images with very high frequency. Most suitable localization algorithm in this case is You Only Look Once (YOLO) convolutional neural network. YOLO was first introduced in 2016 where object detection was reframed as a single regression problem straight from image pixels to bounding box and class probabilities [9]. This made sure that YOLO needed only one passage through the network to obtain localization result which made it much faster then existing algorithms which were based on region proposals. Over time, a number of object detectors based on YOLO were developed. Most notably, YOLO9000 [28], YOLOv3 [29] and YOLOv4 [30] and YOLOv8 [31]. This article uses YOLOv4 since it is latest available in Matlab. YOLOv4 in Matlab is pretrained on Common Objects in Context (COCO) dataset [32] which consists of 80 classes and over 330000 images.

6. Camera model

Let us assume the pinhole camera model as is shown in the Figure 4 [33].

Let \( X \) and \( Y \) denote the object size and let \( x \) and \( y \) denote size of the image in the image plane. Let \( f_x \) and \( f_y \) denote horizontal and vertical focal length measured in pixels which is also the distance of image plane from the CCD sensor. Inspecting Figure 4, it can be seen that tracking error angle in vertical plane can be evaluated as follows:

\[
\epsilon_z = \arctan \frac{y}{f_y}
\]

(18)

For vertical tracking error angle, the equation is analogous:

\[
\epsilon_y = \arctan \frac{x}{f_x}
\]

(19)

It is straightforward to implement calculation of the tracking errors. YOLO detector can obtain bounding box around the detected target with coordinates of the upper left corner (denoted with \( x_t \) and \( y_t \)) and its width and height (denoted with \( w \) and \( h \)). Now expressions for vertical and horizontal tracking errors are as follows:

\[
\epsilon_z = \arctan \frac{y_t + h/2}{f_y}
\]

(20)

\[
\epsilon_y = \arctan \frac{x_t + w/2}{f_x}
\]

(21)

7. Seeker camera gimbal servo controls

The camera is mounted on the head of the missile and it can rotate about the vertical and the horizontal axes on gimbal (yaw, pitch) using two DC motors. Rotational motions of these two motors is independent. Figure 5 shows mechanical structure of a missile seeker.

The goal of these DC motor based servo systems is to nullify the tracking error so that the target is always at the center of the obtained image. Therefore, the target tracking problem can be viewed as a control problem where the tracking error is considered an output value with desired reference value equal to zero. This way, radome refraction error is minimized and better measurements are available. It is important to note that the gimbal servo control systems have to be sufficiently fast in order to quickly eliminate tracking errors. The servo motor can be modeled as a second order linear system as follows:

\[
G(s) = \frac{\omega_n^2}{s(s + 2\xi\omega_n)}
\]

(22)

Where \( \xi \) and \( \omega_n \) are damping coefficient and system angular frequency respectively. The tracking loop dynamics bandwidth is very wide, usually about 100 rad/sec [10]. A standard PID controller is used for each gimbal axis. The controllers are tuned so that they achieve acceptable settling time and overshoot.

8. Missile guidance system synthesis

Each previous section presents a single part of the missile guidance system. The typical missile guidance system consists of guidance system, missile seeker, autopilot and
the missile itself. Figure 6 shows the block diagram of the missile guidance system.

![Block Diagram](image)

Figure 6: Missile guidance system design

Missile guidance systems design showed in the figure 6 only measures the image of the target. The remaining measurements can be obtained from the missiles internal sensors. Missile linear and angular velocities can be measured using inertial measurement units (IMU), from which Euler angles and their derivatives can be obtained using sensor fusion technique. Gimbal angles are measured by appropriate angle transducers.

9. Simulation

Previously described missile guidance system was implemented in Matlab and Simulink implementing previously described equations. Missile and the target position and orientation are fed into Unreal Engine blocks. Matlab offers Simulation 3D camera block to obtain images from the Unreal Engine scene. Simulation 3D camera block outputs an image which is fed to the YOLO block (implemented as a Matlab system) which calculates tracking errors given by equations 18 and 19. Camera generates 100 frames per second (fps) which is fast enough to capture tracking error dynamics. Obtained tracking errors are then fed into two PID controllers as error inputs. Controller outputs are fed into gimbal systems as inputs and gimbal positions are fed into camera as inputs to rotate the camera relatively to the missile body. Based on the equation 15, the seeker subsystem estimates the line of sight angular rate in the vertical and the horizontal plane. Subsystem for proportional navigation calculates the required lateral accelerations in vertical and horizontal plane. The autopilot subsystem calculates required elevator, rudder and aileron deflections.

Figure 7 shows trajectories of the target and the missile during the terminal guidance phase. The target is at the initial relative height of 200 meters and an initial relative distance of 500 meters. The target is escaping with the velocity 100 m/s and moving sideways with the velocity 20 m/s. It can also be seen that the missile is aiming at future intercept point rather than following the target. This means that PN requires lower commanded acceleration comparing to more traditional guidance methods.

Figure 8 shows one frame which is processed by YOLO detection algorithm. Here it can be seen that YOLO finds the bounding box around the object and that the object is in the center of the image.
Figure 9: Tracking errors

Figure 10: Estimated LOS rate in vertical plane

Figure 11: Estimated LOS rate in horizontal plane

Figure 12: Missile Euler angles

Figure 13: Missile vertical lateral acceleration

Figure 14: Missile horizontal lateral acceleration
Table 2: Missile performance table

<table>
<thead>
<tr>
<th>Initial yaw heading error</th>
<th>Initial pitch heading error</th>
<th>Target horizontal acceleration</th>
<th>Target vertical acceleration</th>
<th>Miss distance</th>
<th>Maximum required horizontal acceleration</th>
<th>Maximum required vertical acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>−0.15 rad</td>
<td>0.15 rad</td>
<td>0</td>
<td>0</td>
<td>0 m</td>
<td>1.0384 m/s²</td>
<td>9.81 m/s²</td>
</tr>
<tr>
<td>−0.15 rad</td>
<td>0 rad</td>
<td>0</td>
<td>0</td>
<td>0 m</td>
<td>1.2325 m/s²</td>
<td>9.6998 m/s²</td>
</tr>
<tr>
<td>0 rad</td>
<td>0.15 g</td>
<td>0</td>
<td>0</td>
<td>0 m</td>
<td>1.8232 m/s²</td>
<td>43.6711 m/s²</td>
</tr>
<tr>
<td>0 rad</td>
<td>0.3 g</td>
<td>0</td>
<td>0</td>
<td>0 m</td>
<td>3.692 m/s²</td>
<td>43.6711 m/s²</td>
</tr>
<tr>
<td>0 rad</td>
<td>0.45 g</td>
<td>0</td>
<td>0</td>
<td>2.7509 m</td>
<td>5.4415 m/s²</td>
<td>43.6711 m/s²</td>
</tr>
<tr>
<td>0 rad</td>
<td>0.6 g</td>
<td>0</td>
<td>0.15 g</td>
<td>0 m</td>
<td>7.2246 m/s²</td>
<td>43.6562 m/s²</td>
</tr>
<tr>
<td>0 rad</td>
<td>0.3 g</td>
<td>0</td>
<td>0.3 g</td>
<td>0 m</td>
<td>22.7714 m/s²</td>
<td>43.6562 m/s²</td>
</tr>
<tr>
<td>0 rad</td>
<td>0.45 g</td>
<td>0</td>
<td>0.15 g</td>
<td>0 m</td>
<td>23.2802 m/s²</td>
<td>43.6562 m/s²</td>
</tr>
<tr>
<td>0 rad</td>
<td>0.6 g</td>
<td>0</td>
<td>0.15 g</td>
<td>13.3609 m</td>
<td>23.2802 m/s²</td>
<td>43.6562 m/s²</td>
</tr>
</tbody>
</table>

10. Performance analysis

Proportional navigation can guarantee interception of a target moving with constant velocity. If the target is maneuvering or accelerating, there can be nonzero miss distance. Proportional navigation can ensure interception even if there is an initial heading error, although this requires the missile to have more energy available and larger demanded accelerations. Table 2 shows performance for various heading errors and target accelerations. Final miss distance and maximum required accelerations are chosen as a measure of performance. It can be seen that proportional navigation can nullify the miss distance due to the initial heading error given that the missile can achieve demanded lateral acceleration. If the target is accelerating, the proportional navigation may not be able to nullify the miss distance.

11. Conclusion

This article presents design and simulation of missile homing system. The missile homing system consists of guidance law, autopilot and imaging based target tracking subsystem (missile seeker). In this work the guidance law is based on true proportional navigation. Autopilots stabilize roll angle and regulate horizontal and vertical accelerations. They are standard linear multi loop controllers. Two axes (pitch, yaw) gimbaled seeker use standard PID controllers to track the target. The seeker uses state-of-art deep machine learning YOLO (You Only Look Once) object detector for tracking error measurements and LOS rate estimation. Comprehensive homing missile simulator is developed, and simulation results show robust accuracy of the proposed scheme to target velocity, launch heading errors and low level target maneuvering. Our future research work will focus on design and simulation of advanced guidance and control laws, as well as design and simulation of advanced machine learning algorithms.

References


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