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Editorial

As the digital and analytical frontiers of research continue to expand, the integration of emerging technologies with robust quantitative methodologies becomes essential to support sustainable progress. In this issue, the featured studies delve into two distinct yet impactful areas: the business strategies surrounding 5G network deployment in Sub-Saharan Africa, and statistical innovation in population estimation through two-phase stratified sampling. These contributions demonstrate the relevance of context-driven technological planning and methodological precision in addressing current and future societal needs.

The rollout of 5G networks across Sub-Saharan Africa marks a critical juncture for mobile network operators (MNOs) seeking to harness digital transformation for economic growth. In a region undergoing rapid technological adoption, the focus shifts beyond infrastructure to business model adaptation. By quantitatively analyzing the key components of MNOs' 5G business strategies, this study highlights the interconnected influence of different model elements on overall performance. With economic disruption and increased market complexity accompanying the 5G transition, the insights offered here support a recalibration of existing models to ensure long-term viability. The empirical approach uncovers both direct and indirect dependencies within business model frameworks, providing a data-driven foundation for strategic decision-making in the evolving telecommunications landscape [1].

Precision in statistical estimation plays a foundational role in reliable data interpretation, especially in complex sampling frameworks. This research contributes to the field by introducing a refined estimator for population mean estimation in two-phase stratified sampling, utilizing exponential and logarithmic transformations. By addressing bias and mean squared error (MSE) up to the first degree of approximation, the proposed method achieves superior performance compared to traditional and contemporary estimators such as $\bar{y}_{ds}$, the Ige & Tripathi ratio estimator, and the Tailor et al. exponential estimator. Through rigorous theoretical derivation and simulation-based validation, the new estimator demonstrates enhanced accuracy and efficiency, reinforcing its applicability in practical survey analysis and statistical modeling [2].

Together, these studies reinforce the dynamic nature of contemporary research, where innovative solutions are applied to both technological and methodological domains. Whether by redefining digital business frameworks or advancing statistical estimators, these works offer valuable pathways for researchers, practitioners, and policymakers seeking to align innovation with real-world challenges.

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- [1] L. Banda, "Analysis of 5G Business Model Components for Mobile Network Operators in Sub-Saharan Africa," *Journal of Engineering Research and Sciences*, vol. 4, no. 2, pp. 1–10, 2025, doi:10.55708/js0402001.
- [2] S. Malik, R. Sharma, R. Gupta, "Product in Product Type Estimator with Exponential and Log Function to Estimate Population Mean Using DSS," *Journal of Engineering Research and Sciences*, vol. 4, no. 2, pp. 11–17, 2025, doi:10.55708/js0402002.

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Analysis of 5G Business Model Components for Mobile Network Operators in Sub-Saharan Africa

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ABSTRACT: The fierce race among mobile network operators (MNOs) to roll out fifth-generation (5G) networks has intensified. One of the potential markets where 5G deployment has increased tremendously is Sub-Saharan Africa. This is primarily due to rapid economic growth and the new opportunities that 5G networks and the associated technologies are expected to offer through sustainable digital transformation. However, research on 5G has mainly focused on the technical aspects with minimal consideration of the business side. Furthermore, research on business models for emerging 5G networks in the African context has received little attention from the scholarly community. This article explores and analyzes the components of the 5G business model for mobile network operators in Sub-Saharan Africa. This study is timely as MNOs should re-evaluate their existing business models to withstand the economic disruption that 5G networks bring. The study was conducted using a quantitative research method through a statistical analysis approach. Empirical results show that the performance of some business model components directly affects others, while other components are not directly related.

KEYWORDS: 5G, Business Model, Components, Mobile Network Operator, Sub-Saharan Africa

1. Introduction

Fifth generation mobile networks (5G) are part of the next-generation in the evolving series of mobile networks that have been defined and standardized by the International Telecommunication Union (ITU) as IMT-2020 [1]. 5G is considered a transformative technology with improved technical characteristics compared to previous generations of mobile technologies and can support the development of innovative service applications across the economy [2]. Consequently, 5G is expected to contribute a total of approximately US\$22.2 trillion to global GDP and US\$588 billion in global tax revenue during the period 2020 to 2034 [3]. Furthermore, 5G networks and the underlying technologies offer a new opportunity to drive economic growth in emerging economies such as sub-Saharan Africa [4].

One of the major economic disruptions that has resulted from the introduction of 5G networks is the complete change in the design and implementation of mobile network operators (MNOs) business models. To

this end, 5G networks require novel business models that can deliver both economic value and technological innovation to MNOs in a sustainable manner. This is crucial as MNOs should have the flexibility to rethink their existing business models in order to remain competitive and viable given the economic peculiarities that 5G networks bring [5].

Research on 5G business models has focused more on the conceptual framework approach without going into the details of the business model components and their interrelationships. This article examines the business model component for 5G network operators. Although a global perspective is regarded as an appropriate approach, the MNOs considered in this study come from emerging economies, particularly the Sub-Saharan Africa region. Therefore, this study occupies a unique place in the body of knowledge as it analyses components of the 5G business for MNOs in Sub-Saharan Africa. The key research question of the study is:

How can the components of the 5G business model be analysed to have a positive impact on the business viability of mobile network operators in Sub-Saharan Africa?

The rest of the article is structured as follows. Section 2 reviews the related work in the existing literature. Section 3 highlights 5G networks from an African perspective. Section 4 introduces the components of the 5G business model and their relationships. Section 5 discusses the research methodology while Section 6 provides the results, analysis and discussion. Section 7 finally concludes the article.

2. Related Work

Several recent developments in 5G business models for mobile network operators have been published extensively in the existing literature. A survey article on 5G business models for mobile network operators was presented in [3]. The authors present various business models that apply to both public and private 5G mobile networks. However, no data analysis was carried out in the work as the work was purely a survey study. A techno-economic analysis of private 5G networks was conducted in [6]. The authors focused on the cost structures associated with the deployment of private 5G networks. Nonetheless, other components of the value chain such as value proposition, customer segment and infrastructure management were not taken into account. Furthermore, the work in [6] was only applicable to private networks.

In [7], a conceptual business model framework for AI-based private 5G-IoT networks was presented. The author argues that the study contributes to the development of innovative business model solutions for 5G, Internet of Things (IoT) and Artificial Intelligence (AI). However, the work in [7] was only at a theoretical level and did not include data collection and analysis. Additionally, an analytical framework was developed in [8] that aimed to calculate the revenue of mobile operators while ensuring end-users' service satisfaction. The problem of revenue maximization is solved algorithmically through proposed green energy operation and allocation strategies. However, the work in [8] was only limited to the revenue aspects of the business model concept. Moreover, other business model elements such as cost structure, value proposition, customer segments and infrastructure management were not considered.

In [9], 5G network slicing business models were presented. The outlined business models target the Internet of Vehicles (IoV) and the Internet of Things (IoT) for maritime vertical applications. The authors' focus was on generic business model approaches and did not go into the analysis of the business model components. A study in [10] identified six key topics for the techno-economic assessment of 5G, including business models, use cases, technologies, modeling techniques, financial indicators and other specific focus areas. However, the work in [10]

only considered theoretically formulated business models without including the practical implementation aspects by key players such as mobile network operators, industry verticals, regulators and policymakers.

This study approached the business model concept from a strategic management perspective by focusing on value proposition, value creation, value delivery and value capture activities within the business ecosystem of 5G mobile network operators [11]. The value chain activities can be translated into the four business model components, namely 5G services, 5G infrastructure management, 5G customer interface and 5G financial aspects.

3. 5G Networks in Africa

3.1. 5G Opportunities

5G provides a myriad of opportunities for Africa to actively participate and play a critical role in the continued global development and deployment of these emerging technologies. This is despite Africa having historically lagged behind in technological advancements [12]. The introduction of 5G in Africa promises various socio-economic benefits such as job creation, growth in income per household, reduction in carbon emissions and improved quality of life for inhabitants [13]. Furthermore, 5G can enable a long-term digital transformation and contribute to the emergence of sustainable digitalized societies [14]. 5G will offer greater access to higher broadband capacity and reliable connectivity for the creation of a digitally connected smart society needed to realize the UN 2030 Agenda and Sustainable Development Goals (SDGs) at national and regional levels [15].

3.2. 5G Deployment and Adoption Forecast

According to [16], there will be 226 million 5G connections in Sub-Saharan by 2030, and this will represent a regional adoption rate of 17%. However, the penetration rate of 5G networks in Africa is still minimal. For instance, [17] observed that as of 2022, the adoption rate of 5G was approximately 1% of all mobile connections and is expected to increase to 8% by 2026 and reaching 22% by 2030. The 5G transport network is mainly supported by fiber infrastructure due to the higher transmission bandwidth and low transmission latency of fiber optic technology [18]. According to [19], the fiber penetration rate in Africa was 10% in 2017 compared to 30% globally and will only reach 15% by 2025 compared to 40% globally.

Despite the ongoing massive deployment of 5G technologies in the Sub-Saharan African region, the GSMA report in [17], predicts that 4G will remain a dominant technology for the foreseeable future before 5G gains maturity. Figure 1 shows the scope of mobile

networks in Africa from 2020 to 2030 as forecast by the GSMA report in [17].

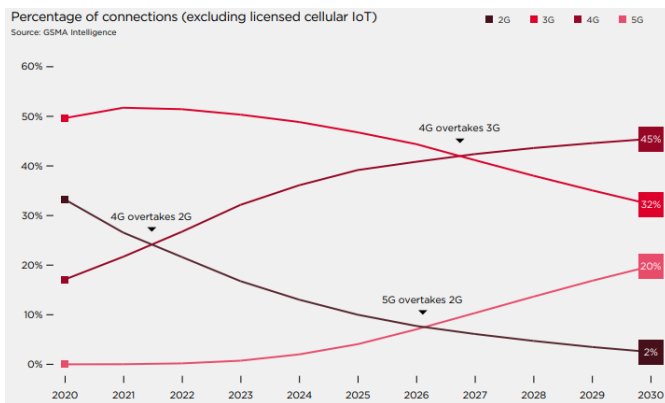


Figure 1: Scope of mobile networks in Africa from 2020 to 2023 [17]

3.3. 5G Rollout Challenges

The extensive rollout of 5G in most African markets has been impeded by various technical, socio-economic and societal challenges. For example, the deployment 5G in certain regions was inhibited by speculations of negative health consequences on society by linking 5G networks with the spread of the COVID-19 pandemic [20]. Another key barrier to the deployment of 5G is the huge capital expenditure required by African mobile network operators to upgrade or purchase 5G network equipment from suppliers [13]. Other challenges facing mobile network operators in rolling out 5G have been outlined in [3]. These include exorbitant spectrum pricing by national regulators, unavailability and unaffordability of 5G-supporting devices, inadequate or substandard 5G network infrastructure, low fiber penetration for 5G backhaul network, and unreliable power supply in most African countries.

4. 5G Business Model Components

4.1. 5G Value Chain

In order to successfully develop and implement viable and sustainable 5G business models, mobile network operators must precisely define the composition

of the 5G value chain. In this study, the proposed 5G value chain elements are based on the business model ontology described in [21]. The 5G value chain activities include the following process.

Value proposition: A summary of the benefits a mobile operator offers its customers and how those benefits differ from the competition.

Value creation: The process of generating 5G services that are more valuable than the resources used to create them.

Value delivery: The process that ensures that the value created reaches the intended customer base.

Value capture: The process of retaining a portion of the value created by a mobile operator and converting it into profit. Direct value capture comes from the direct use of 5G services by subscriber customers, indirect value capture arises from the use of infrastructure services by third parties.

4.2. Business Model Components and their Relationships

The proposed 5G business model for mobile network operators consists of four main components: 5G services, 5G infrastructure management, 5G customer interface and 5G financial aspects. These components are interconnected through the 5G value chain activities described previously.

(i) 5G Services

The value proposition envisages a wide range of products and services that a mobile operator offers. The three main 5G service classes that form the value proposition component of the proposed conceptual business model framework are: (i) *enhanced mobile broadband (eMBB)* services tailored for throughput-sensitive applications such as high-speed broadband access, immersive mobile services, and high-quality video content applications; (ii) *massive machine type communication (mMTC)* services target applications with high device connectivity density such as industrial IoT and commercial IoT service applications; and (iii) *ultra*

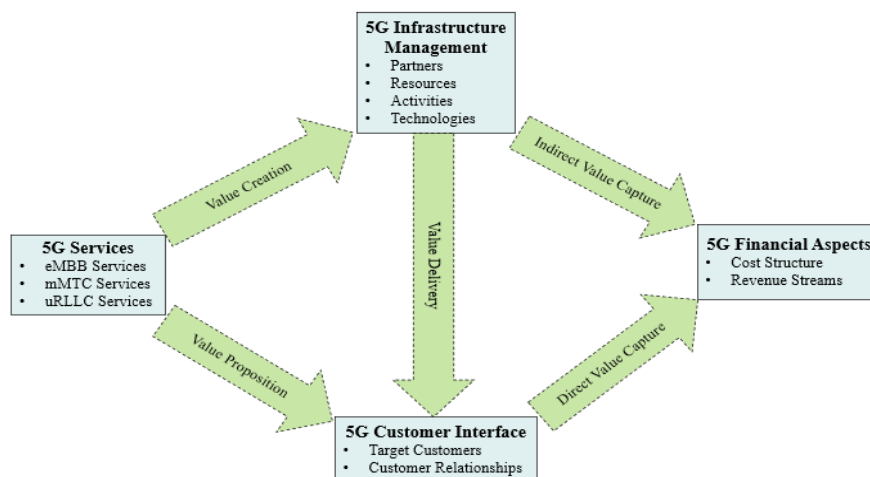


Figure 2: 5G business model components and their interrelationships across value chain activities

reliable low latency communications (uRLLC) services intended for low latency (delay) and high reliability communications, for example mission-critical applications such as industrial automation and remote surgery.

(ii) 5G Infrastructure Management

Infrastructure management describes the technology design, network infrastructure and resources available to a mobile operator to deliver the relevant proposed value to intended customers. The infrastructure management component of the proposed conceptual business model framework consists of four sub-components: (i) *5G partners*, highlighting the need for collaborative arrangements with external organizations to effectively deliver and monetize the proposed value. This includes equipment vendors, device manufacturers, platform providers, regulatory and policymakers and other mobile operators; (ii) *5G resources* which describe the resources available to the mobile operator to ensure that value is created and delivered in line with business objectives. Examples include physical, intellectual, human, financial, spectrum and site equipment and infrastructure; (iii) *5G activities* which describe the activities and competencies required to implement the mobile operator's business model. This includes managing partnerships, network design and maintenance, mobile service delivery and technical customer support; and (iv) *5G technologies*, which describe the underlying and supporting technologies that a mobile operator must design and implement to ensure that 5G services are created and delivered to intended customers. Technologies include network slicing, green communications, digital platforms, SDN and NFV, ultra-densification, IoT and D2D communications, and massive MIMO and beamforming.

(iii) 5G Customer Interface

The customer interface component of the proposed 5G business model consists of three sub-components including: (i) *Customer segment*, which describes the customer base to which a mobile operator wants to offer value, i.e., customers attracted by the value proposition. Examples include state agencies, local municipalities, private enterprises and vertical industries; (ii) *Customer relationships* which explain the nature of the connections a mobile operator creates between itself and its different customer segments, such self-services, personal assistance services, automated services and community-based services.

(iv) 5G Financial Aspects

The financial aspects of the proposed 5G business model describe a mobile operator's ability to capture and monetize the proposed value for intended customers. The financial aspects consist of two elements: (i) *Cost structure*, which summarizes the monetary impact of the resources used in the business model such as fixed costs, variable

costs, CapEx and OpEx; and (ii) *Revenue streams*, which describe how a mobile operator makes money through a variety of revenue flows such as subscription fees, leasing/rental of assets, advertising and revenue sharing through partnerships.

Figure 2 above illustrates the components of the proposed 5G business model and their interrelationships across value chain activities.

5. Methodology

5.1. Research Design

The study was conducted in two phases using quantitative research method outlined in [22]. The two phases of the research design are shown in Figure 3.

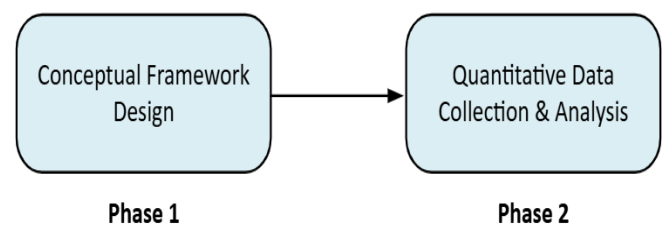


Figure 3: Two-phased research design

Phase 1: Results from the reviewed literature are used to design a conceptual business model framework that 5G mobile network operators can implement.

Phase 2: The conceptualised business model framework is subjected to statistical testing using quantitative methods.

5.2. Conceptual Framework

The proposed conceptual business model framework of this study is shown in Figure 4. Furthermore, based on the empirical literature reviews, the study postulates six research hypotheses which are outlined below.

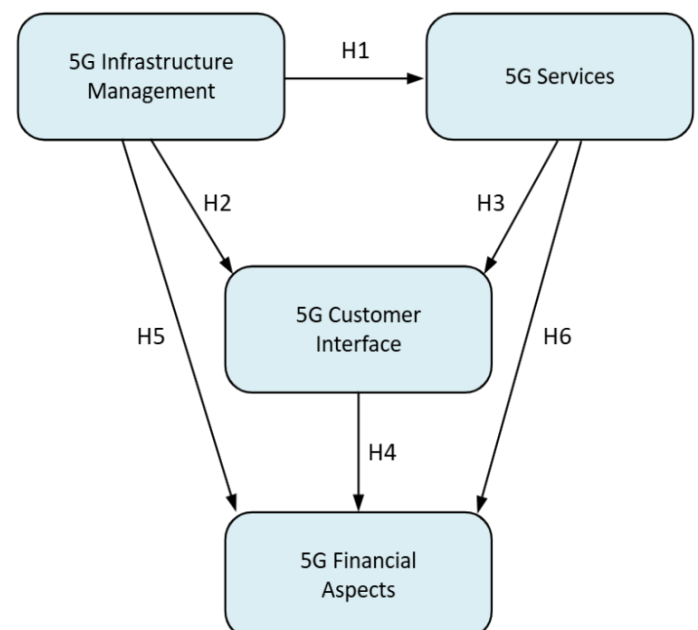


Figure 4: Formulated conceptual framework design

Hypotheses definitions:

H1: There is a significant positive impact of 5G Infrastructure Management on 5G Services.

H2: There is a significant positive impact of 5G Infrastructure Management on the 5G Customer Interface.

H3: There is a significant positive influence of 5G Services on the 5G Customer Interface.

H4: There is a significant positive influence of 5G Customer Interface on the 5G Financial Aspects.

H5: There is a significant positive impact of 5G Infrastructure Management on 5G Financial Aspects .

H6: There is a significant positive influence of 5G Service on 5G Financial Aspects.

5.3. Data Collection Procedures

This study involved quantitative data collection through structured online survey questionnaires. Qualtrics, an online data collection software was used to design, capture and summarize data. A total of 102 questionnaires were distributed for the study. There were 71 responses. However, only 62 responses were found to be usable for data analysis. A summary of the sample response rate is shown in Table 1.

Table 1: Data Collection Sample Response Rate

Questionnaire distributed	102
Questionnaires returned	71
Usable questionnaires	62
Usable response rate	60.8%

6. Results, Analysis and Discussions

The data analysis of the study was conducted in two phases using statistical analysis. In the first part, descriptive statistics are provided for both demographic data and the measurement instruments using Statistical Package for the Social Sciences (SPSS) software. The second part provides results for hypothesis testing using structural equation models (SEMs) to derive various coefficients using SmartPLS software.

6.1. Demographic Descriptive Statistics Results

Demographic descriptive results are based on descriptive statistical analysis which was conducted in Statistical Package for SPSS) software. Demographic data included participants' gender, position in the company, department/functional area and years of experience in the mobile industry. SPSS software was used to clean data and present results using frequency tables.

(i) Research participants by gender

Demographic data results by participants' gender show that 67.7% of the participants were males while 32.3% of the participants were females. Table 2 shows the sample distribution according to participants' gender.

Table 2: Research Participants by Gender

	Frequency	Cumulative Frequency	Valid Percent	Cumulative Percent
Valid Male	42	67.7	67.7	67.7
Female	20	32.3	32.3	100
Total	62	100	100	

(ii) Research participants by position in the company

The results of the demographic data by participants' positions in the company show that the majority of participants were in middle management (32.3%), while top executives made up only 12.9% of the data collected. Table 3 shows the sampling distribution of research participants according to their position in the company.

Table 3: Research Participants by Position in the Company

	Frequency	Cumulative Frequency	Valid Percent	Cumulative Percent
Valid Top Executive	8	8	12.9	12.9
Senior Manager	13	21	21.0	33.9
Mid-level Manager	20	41	32.3	66.2
Employee	18	59	29.0	95.2
Others	3	62	4.8	100
Total	62		100	

(iii) Research participants by functional area

Responses by functional area within the company indicate that most participants came from the networks/technical area (69.4%), followed by the information technology (IT) area (17.7%). Other functional areas such as finance, sales/marketing and logistics were hardly represented. Table 3 shows the demographic distribution of participants by functional area in the company.

Table 4: Research Participants by Functional Area

	Frequency	Cumulative Frequency	Valid Percent	Cumulative Percent
Valid Networks/Technical	43	43	69.4	69.4
I.T	11	54	17.7	87.1
Finance	2	56	3.2	90.3
Sales/Marketing	4	60	6.5	96.8
Logistics	1	61	1.6	98.4
Others	1	62	1.6	100
Total	62		100	

(iv) Research participants by years of experience

The participants in the study stated their years of experience in the mobile communications industry. Sample data shows that most participants had more than 10 years of experience (62.9%). Those with 6 to 10 years of experience accounted for 24.2%, while those with 1 to 5 years of experience accounted for 12.9% of the total sample response. Table 4 shows the sample response of years of experience in the mobile industry.

Table 5: Research Participants by Years of Experience

	Frequency	Cumulative Frequency	Valid Percent	Cumulative Percent
Valid 1 - 5 years	8	8	12.9	12.9
6 - 10 years	15	23	24.2	37.1
Over 10 years	39	62	62.9	100
Total	62		100	

6.2. PLS-SEM Inferential Statistics Results

Structural equation modeling (SEM) is a diverse set of methods that scientists use when conducting quantitative business research [23]. Partial Least Square SEM (PLS-SEM) is a form of SEM used to maximize the variance of the dependent variable and is often applied when working with complex models, limited sample sizes, and the need for prediction [24]. Due to the small sample size and complexity of this study, PLS-SEM was selected.

The PLS-SEM analysis is conducted in two phases [25]:

(1) *Measurement model analysis*, which specifies relationships between research constructs and research indicators, and (2) *Structural model analysis*, which specifies the relationships between research constructs. In this study, PLS-SEM analysis was performed using SmartPLS software, from which various measurement coefficients were derived. The resulting coefficients were used to test the formulated research hypothesis.

The PLS-SEM in the SmartPLS software shows the measurement indicators (measurement variables) and constructs (latent variables), which can be either independent or dependent. One or more measurement indicators are assigned to each latent variable. Measurement indicators correspond to questions in a research questionnaire, while constructs are derived from a conceptual or theoretical model. Independent variables are constructs that are not dependent on other constructs, while dependent variables are those that are dependent on one or more constructs. The relationship between variables is called a path and is associated with a path coefficient. In this study, 5G Infrastructure Management (5G-IM) was the only independent variable and was supported by two measurement indicators. The dependent variables include 5G Services (5G-S) with two measurement indicators, 5G Customer Interface (5G-CI) with four measurement indicators, and 5G Financial Aspects (5G-FA) with six measurement indicators. Figure 5 shows the PLS-SEM model of this study in SmartPLS software.

(i) Measurement Model Results and Analysis

A reflective measurement model was used in the study and the criteria for evaluating the reflective measurement model include: internal consistency, convergent validity and discriminant validity [26]. Internal consistency is a quality criterion of a construct that requires a high degree of correlation between indicators of a particular construct [27]. The two common measures of internal consistency are Cronbach's alpha and composite reliability. Convergent validity is the extent to which a measure is positively correlated with an alternative measure of the same construct and the measurement criterion is the average variance extracted (AVE) [27]. Discriminant validity is the extent to which a construct actually differs from other constructs by empirical standards and is commonly measured using the heterotrait-monotrait ratio (HTMT) [28]. Table 5 shows the results of the measurement model, including the threshold value of the measurement criteria and the conclusions on the results.

(ii) Structural Model Results and Analysis

Assessment of the structural model is the second phase of the PLS-SEM approach, which follows confirmation of the reliability and validity of the construct measures. The structural model and its latent variables represent the stable, theoretically and conceptually established contextual relationship between observed data on the input and output sides of the PLS-SEM model [27]. Structural modelling is used to assess causal relationships between latent variables and includes multiple regression analysis and path analysis [29]. Table 6 shows the significance results of the path coefficient with the associated parameters: t -values, p -values, 95% confidence intervals and the standard deviation. According to [28], p -values below 0.05 and t -values above 1.96 indicate path significance and a strong relationship between two variables.

(iii) Hypothesis Testing

After the assessment of the measurement and structural models of the PLS-SEM analysis was completed, the study examined the relationships between research constructs through path analysis and hypothesis testing. When testing hypotheses, path coefficients (β) reflect the nature and strength of the relationships between variables: the higher the β value, the stronger the relationship. To determine whether the hypotheses are supported or not, the β -values, p -values and t -values are analysed within a 95% confidence interval [28]. For the hypothesis to be supported, the following conditions

must be met: $p\text{-value} < 0.05$, $t\text{-value} > 1.96$ and 95% confidence interval. Table 7 shows the hypothesis testing results of the current study. From the results of Table 12, four of the six hypotheses are significant and supported. Further analysis is carried out as follows:

H1: There is a significant positive impact of 5G Infrastructure Management (5G-IM) on 5G Services (5G-S). H1 evaluates whether 5G-IM significantly and positively affects 5G-S. The results revealed that 5G-IM has a significant and positive impact on 5G-S ($\beta = 0.539$, $t = 7.537$, $p < 0.05$). Hence H1 was supported.

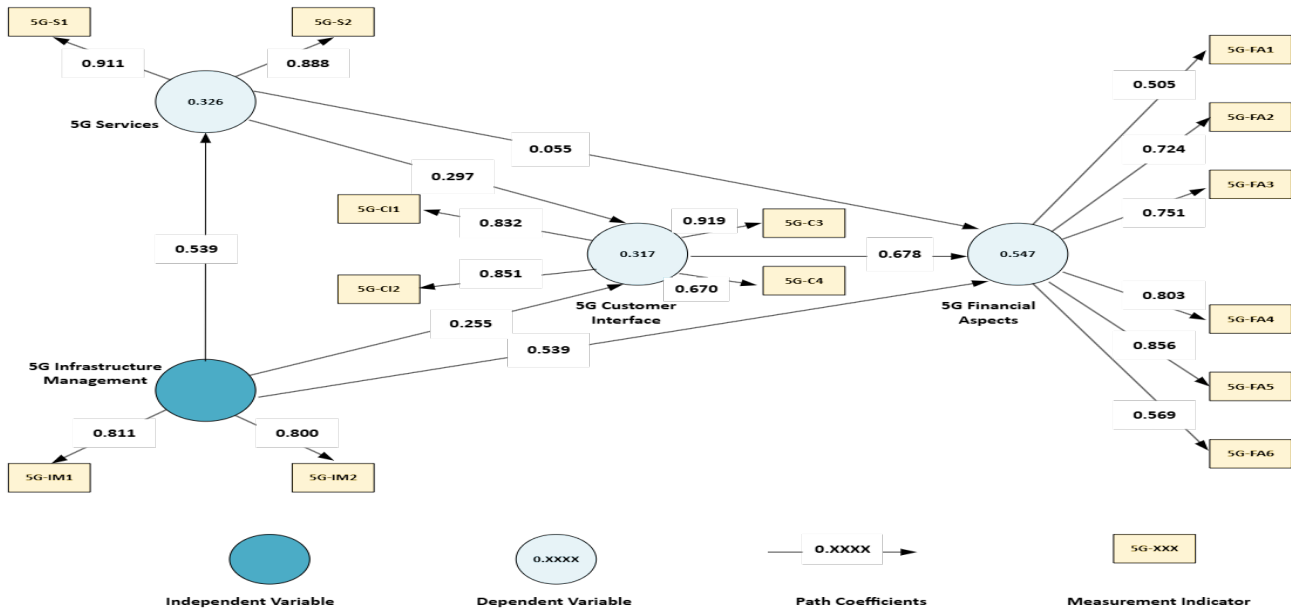


Figure 5: PLS-SEM model in SmartPLS software

Table 5: Results from the Measurement Model Evaluation

Research Construct	Internal Consistency				Convergent Validity		Discriminant Validity		
	Cronbach Alpha (CA) Threshold (CA > 0.7)		Composite Reliability (CR) Threshold (CR > 0.7)		Average Variance Extracted Threshold (AVE < 0.5)		Heterotrait-monotrait ratio (HTMT) Threshold (HTMT < 0.9)		
	Value	Conclusion	Value	Conclusion	Value	Conclusion	Constructs Relationship	Value	Conclusion
5G Infrastructure Management (5G-IM)	0.715	Acceptable	0.787	Acceptable	0.649	Acceptable	5G-IM ==> 5G-S	0.810	Acceptable
5G Services (5G-S)	0.765	Acceptable	0.902	Acceptable	0.809	Acceptable	5G-IM ==> 5G-CI	0.657	Acceptable
5G Customer Interface (5G-CI)	0.836	Acceptable	0.892	Acceptable	0.678	Acceptable	5G-IM ==> 5G-FA	0.605	Acceptable
5G Financial Aspects (5G-FA)	0.798	Acceptable	0.857	Acceptable	0.508	Acceptable	5G-S ==> 5G-CI	0.548	Acceptable
							5G-S ==> 5G-FA	0.440	Acceptable
							5G-CI ==> 5G-FA	0.849	Acceptable

Table 6: Results from the Structural Model Evaluation

	Path Coefficient	Standard Deviation	t- Values	p-Values	95% Confidence Intervals	Significance ($p < 0.05$)
5G Services ==> 5G Customer Interface	0.297	0.144	2.063	0.042	[0.026, 0.502]	Yes
5G Services ==> 5G Financial Aspects	0.055	0.135	0.405	0.686	[-0.209, 0.251]	No
5G Customer Interface ==> 5G Financial Aspects	0.678	0.084	8.110	0.000	[0.511, 0.812]	Yes
5G Infrastructure Management ==> 5G Services	0.539	0.071	7.537	0.000	[0.433, 0.696]	Yes
5G Infrastructure Management ==> 5G Customer Interface	0.255	0.115	2.227	0.028	[0.067, 0.473]	Yes
5G Infrastructure Management ==> 5G Financial Aspects	0.059	0.12	0.491	0.624	[-0.179, 0.271]	No

Table 7: Results from Hypothesis Test

Path Hypothesis	Path Coefficient (β)	t- Values	p-Values	95% Confidence Intervals	Significance ($p < 0.05$)	Test Results
H1: 5G Infrastructure Management => 5G Service	0.539	7.537	0.000	[0.433, 0.696]	Yes	Supported
H2: 5G Infrastructure Management =>5G Customer Interface	0.255	2.227	0.028	[0.067, 0.473]	Yes	Supported
H3: 5G Service => 5G Customer Interface	0.297	2.063	0.042	[0.026, 0.502]	Yes	Supported
H4: 5G Customer Interface => 5G Financial Aspects	0.678	8.110	0.000	[0.511, 0.812]	Yes	Supported
H5: 5G Infrastructure Management => 5G Financial Aspects	0.059	0.491	0.624	[-0.179, 0.271]	No	Not supported
H6: 5G Service => 5G Financial Aspects	0.055	0.405	0.686	[-0.209, 0.251]	No	Not Supported

H2: There is a significant positive impact of 5G Infrastructure Management (5G-IM) on the 5G Customer Interface (5G-CI). H2 evaluates whether 5G-IM significantly and positively affects 5G-CI. The results revealed that 5G-IM has a significant and positive impact on 5G-CI ($\beta = 0.255$, $t = 2.227$, $p < 0.05$). Hence H2 was supported.

H3: There is a significant positive influence of 5G Services (5G-S) on 5G Customer Interface (5G-CI). H3 evaluates whether 5G-S significantly and positively affects 5G-CI. The results revealed that 5G-S has a significant and positive influence on 5G-CI ($\beta = 0.297$, $t = 2.063$, $p < 0.05$). Hence H3 was supported.

H4: There is a significant positive influence of 5G Customer Interface (5G-CI) on 5G Financial Aspects (5G-FA). H4 evaluates whether 5G-CI significantly and positively affects 5G-FA. The results revealed that 5G-CI has a significant and positive impact on 5G-FA ($\beta = 0.678$, $t = 8.110$, $p < 0.05$). Hence H4 was supported.

H5: There is a significant positive impact of 5G Infrastructure Management (5G-IM) on 5G Financial Aspects (5G-FA). H5 evaluates whether 5G-IM significantly and positively affects 5G-FA. The results revealed that 5G-IM has an insignificant impact on the 5G-FA ($\beta = 0.059$, $t = 0.491$, $p > 0.05$). Hence, H5 was not supported.

H6: There is a significant positive influence of 5G Services (5G-S) on 5G Financial Aspects (5G-FA). H6 evaluates whether 5G-S significantly and positively affect 5G-FA. The results revealed that 5G-S have an insignificant impact on the 5G-FA ($\beta = 0.055$, $t = 0.405$, $p > 0.05$). Hence, H6 was not supported.

(iv) Final Business Model Framework

From the two-phased quantitative research method and the results of the research hypothesis tests, the final business model framework is shown in Figure 6.

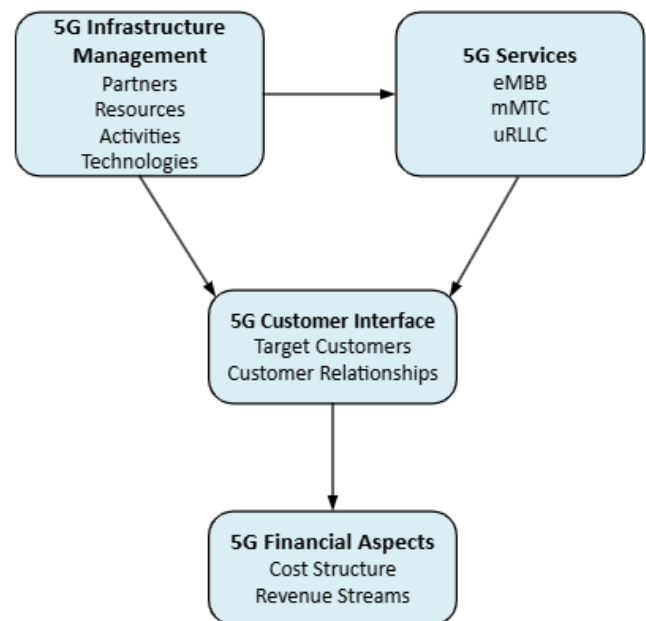


Figure 6: Final Business Model Framework

7. Conclusion

5G networks and associated technologies are expected to drive the digital economy and enable sustainable digital transformation in emerging markets such as Sub-Saharan Africa. However, in order for mobile network operators in these markets to benefit from the economic opportunities of 5G, existing business models should be re-assessed. This article examined and analyzed components of the 5G business model for mobile network operators in Africa. The study was conducted using a quantitative research

method through a statistical analysis approach. Descriptive statistical analyses were performed in SPSS software to obtain demographic results, while inferential statistical analyses were performed using SmartPLS software for PLS-SEM modeling and research hypothesis testing. Of the six research hypotheses formulated, four were significantly supported, while two were insignificant and therefore rejected. Empirical results showed that the performance of some business model components directly affects others, while other components are not directly related. Future work will consider a larger sample size for primary data collection and analysis as more mobile network operators would have deployed 5G networks.

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Product in Product Type Estimator with Exponential and Log Function to Estimate Population Mean Using DSS

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ABSTRACT: To estimate the population mean in two-phase stratified sampling, this paper aims to construct a new product in the product type estimator using exponential and log functions. When using two samples for stratification, this study attempts to solve the equation for bias and mean squared error (MSE) up to the first degree of approximation (fda). The modified estimator is more effective than $\bar{y}_{ds}$ (the usual unbiased estimator), $\hat{Y}_{pp}^{ds}$ (Ige & Tripathi ratio estimator) and $\hat{Y}_{ppe}$ (ratio type exponential estimator) given by theoretically and numerically. Finding out how the proposed estimators stack up against the state-of-the-art is the main motivation for this research. Extensive evaluation of the estimator's performance in a simulated environment. The suggested estimators perform better than other estimators, according to both theoretical and practical studies.

KEYWORDS: Finite population mean, Bias, Mean Squared Error, Double sampling for stratification (DSS)

1. Introduction

Survey research and statistical inference have both seen a rise in the use of double sampling due to its efficiency and adaptability. Double sampling provides a potent method for enhancing the precision and dependability of population estimates within the framework of stratified sampling. Researchers may take advantage of stratification while addressing any constraints and uncertainties via twofold sampling, which involves two consecutive samples.

Stratified sampling is a popular technique that divides a population into several subgroups, or strata, that have common traits. Although stratification may improve the accuracy of estimates, it often necessitates knowing the strata's sizes, means, and variances in advance. However, such data may be scarce, inaccurate, or unavailable in many real-world contexts.

One practical way to address these issues is through double sampling. Sizes, means, and variances of the strata may be gleaned from an initial sample, sometimes called a preliminary or first-phase sample. Next, the second-phase or primary sample is designed and selected using this information. It is usually optimized for efficiency and accuracy.

Multiple benefits accrue from combining stratification with double sampling, such as:

- Enhanced precision and consistency in population estimations
- Improvements in the accuracy of stratum variance and mean estimations
- Greater adaptability when faced with unknown or sparse background data
- An improvement in the treatment of missing and non-response data

Researchers may create more solid statistical conclusions by combining the benefits of stratification and double sampling to create more effective sample designs.

Steps for Double Sampling in Stratification

- Get the ball rolling by surveying a small starting sample with a preliminary survey. Sizes, means, and variances of the strata are elicited from this sample.
- Sort people into different groups or strata according to important factors like age, gender, wealth, or where they live; this process is called stratification.
- Parameter Estimation for the Stratum: Utilise the sample's initial data to calculate the stratum's sizes, variances, and means.
- Optimizing the Allocation of the Second-Phase Sample: Use the estimated stratum characteristics to

achieve this goal. To reduce the estimator's total variance, finding the ideal sample size for each stratum is necessary.

- Phase Two Sampling: Gather the phase two sample using the optimized sample allocation. This sample is usually more significant than the preliminary one to get more accurate estimations.
- Analyse and estimate population metrics like totals, proportions, and averages using the pooled data from both stages. Draw conclusions and judgments about the population from the data by analyzing it.

Advantages of Double Sampling in Stratification

- One advantage is that it improves accuracy. Since sample mistakes are reduced with double sampling, estimates of population parameters are more accurately reflected.
- Second, it's more efficient to use a smaller sample size to get the same level of accuracy when using double sampling since it optimizes sample allocation.
- Improving Uncertainty Handling: Double sampling offers a solution for dealing with population-level Uncertainty and limited historical data.
- Versatility: Double sampling may be used for various populations and sample strategies.
- Cost-Effectiveness: Compared to conventional sampling techniques, double sampling may save money by lowering the needed sample size

1.1. Real-Life Applications of Double Sampling in Stratification

Estimating market size, consumer preferences, and purchasing behavior may be achieved via double sampling. Using a double-sample design, it is possible to estimate demographic variables, including income, education level, and health outcomes; estimates of agricultural revenue, animal numbers, and crop yields may all be made using double sampling. Environmental characteristics, including water and air quality, may be estimated via double sampling. The incidence, prevalence, and risk factors of diseases may be estimated via double-sampling.

Sometimes, the means of auxiliary variate is not known. Under this situation, a DS technique is used. According to this technique, a significant sample is considered to estimate population mean $\bar{X}$. A subsample of assumed size is regarded from the population. In [1], the authors developed estimators for memory type ratios and products using ranked-based sampling methods. In [2], the authors created exponential estimators for ratios and products. In [3], the researchers worked on the method for estimating the yield of cereal experiments is to sample for the grain-to-total-product ratio. In [4], the authors defined an updated stratified random sample estimator that incorporates two more variables. In [5], the

researchers defined a unique Exponential Strategy for the Ratio in Ratio Type Used to Estimate the Mean of a Population. In [6], the authors worked on using supplementary data and doubling for stratification. In [7], the authors developed a estimation of ratios using stratified random sampling models. Similarly [8] gives their work on estimators of the regression type that use the model of double sampling and two auxiliary variables. In [9], the authors defined using stratified random sampling to estimate a ratio product. In [10], the researchers defined a Using the parameters of the auxiliary variables in double sampling, chain ratio type estimators. In [11,12], the authors defined a generic class of stratified weighted population variance estimators that are robust against non-sampling errors. In [13], the authors worked on assessment of the distribution function using calibration techniques. Similarly [14] allows for more accurate ratio estimates when using stratified sampling. In [15], the authors developed a new exponential estimator of finite population mean in stratigraphy using ratio-ratio-type data collected from two samples. In [16], the researchers defined Estimator of limited population mean in stratigraphy using product-product-type exponential sampling with two samples. In [17], a work on assessment of the distribution functions of a population when non-response is present. In [18], authors defined an mean estimators in stratified two-phase sample using exponential ratio and product type.

1.2. Notations

Let the population T of M size such that $T = [T_1, T_2, T_3, \dots, T_M]$. From this population T

- A 1st phase n' sized sample S is drawn using SRSWOR and only auxiliary variate x is observed.
- The S' sample that is stratified into N strata. Let n'_k be the no. of units in k^{th} stratum such that $n' = \sum_{k=1}^N n'_k$
- From each n'_k units, $n_k = u_k n'_k$ sized sample is drawn, where $(0 < u_k < 1)$ is defined as probability of selecting a n_k sized sample which is predetermined from n'_k sized strata and it considered as a S sample of size $n = \sum_{k=1}^N n_k$. In S both y & x are observed. Let y & x be defined as study variate and auxiliary variate respectively.

Where $k = 1, 2, \dots, N$

1.3. Reviewing some previously exist Estimators

By considering study variable as y and auxiliary variables as x and z

$$\bar{X} = \frac{1}{N} \sum_{k=1}^N \sum_{h=1}^{M_k} x_{kh}, \quad \bar{Y} = \frac{1}{M} \sum_{k=1}^N \sum_{h=1}^{M_k} y_{kh}, \quad \bar{Z} = \frac{1}{M} \sum_{k=1}^N \sum_{h=1}^{M_k} z_{kh}$$

$\bar{X}, \bar{Y}$ and $\bar{Z}$ are defined as population mean of x, y and z respectively.

In DSS scheme, the usual unbiased estimator that is defined by

$$\bar{y}_{ds} = \sum_{k=1}^N w_k \bar{y}_k \quad (1)$$

Given by Cochran, classical ratio type estimator that was studied with DSS technique by Ige and Tripathi as

$$\hat{Y}_{PP}^{ds} = \bar{y}_{ds} \frac{\bar{x}'}{\bar{x}_{ds}} \quad (2)$$

where $\bar{x}_{ds} = \sum_{k=1}^N w_k \bar{x}_k$ and $\bar{y}_{ue} = \sum_{k=1}^N w_k \bar{y}_k$ based on 2nd phase sample are unbiased estimators of $\bar{X}$ and $\bar{Y}$ respectively.

By Bahl and Tuteja Ratio-type-exponential estimator of $\bar{Y}$, were given in SRS as

$$\hat{Y}_{PPE} = \bar{y} \exp \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) \quad (3)$$

This estimator was studied in 2014 by Tailor and others in DSS technique as

$$\hat{Y}_{PPE}^{ds} = \bar{y}_{ue} \exp \left(\frac{\bar{x}' - \bar{x}_{ds}}{\bar{x}' + \bar{x}_{ds}} \right) \quad (4)$$

where $\bar{x}' = \sum_{k=1}^{n_k} w_k \bar{x}'_k$ is an unbiased estimator for $\bar{X}$ based on the 1st phase sample.

2. Suggested Novel Estimator

Inspired by Kiregyera a novel product in product-typed estimator with exponential and log function for finite population means is created with the use of DSS approach using known population means of the 2nd auxiliary variable z , as

$$\hat{Y}_{PPS}^{ds} = \bar{y}_{ds} \left[\exp \left(\frac{\bar{x}_{ds} \left(\frac{\bar{z}}{\bar{z}'} \right) - \bar{x}'}{\bar{x}' \left(\frac{\bar{z}}{\bar{z}'} \right) + \bar{x}_{ds}} \right) + \log \left(\frac{\bar{x}' \left(\frac{\bar{z}}{\bar{z}'} \right)^{\alpha_1}}{\bar{x}_{ds} \left(\frac{\bar{z}}{\bar{z}'} \right)^{\alpha_2}} \right) \right] \quad (5)$$

α_1, α_2 are positive reals.

where $\bar{z}' = \sum_{k=1}^{n_k} w_k \bar{z}'_k$ is defined as an unbiased estimator of $\bar{Z}$ on the bases of 1st sample phase.

The bias & MSE of the suggested novel estimator that can be obtained by considering $\bar{y}_{ue} = \bar{Y}(1 + e_0)$, $\bar{x}_{ds} = \bar{X}(1 + e_1)$, $\bar{x}' = \bar{X}(1 + e'_1)$ and $\bar{z}' = \bar{Z}(1 + e'_2)$ such that $E(e_0) = E(e_1) = E(e'_1) = E(e'_2) = 0$ and

$$E(e_0^2) = \frac{1}{\bar{Y}^2} \left[S_y^2 \frac{(1-f)}{n'} + \frac{1}{n'} \sum_{k=1}^N w_k S_{yk}^2 \left(\frac{1}{u_k} - 1 \right) \right]$$

$$E(e_1^2) = \frac{1}{\bar{X}^2} S_x^2 \left(\frac{1-f}{n'} \right)$$

$$E(e_1^2) = \frac{1}{\bar{X}^2} \left[S_x^2 \left(\frac{1-f}{n'} \right) + \frac{1}{n'} \sum_{k=1}^N w_k S_{xk}^2 \left(\frac{1}{u_k} - 1 \right) \right]$$

$$E(e_2^2) = \frac{1}{\bar{Z}^2} S_z^2 \left(\frac{1-f}{n'} \right)$$

$$E(e_0 e_1) = \frac{1}{\bar{Y}\bar{X}} \left[\frac{1-f}{n'} S_{yx} + \frac{1}{n'} \sum_{k=1}^N w_k S_{y x k} \left(\frac{1}{u_k} - 1 \right) \right]$$

$$E(e_1 e'_1) = \frac{1}{\bar{X}^2} S_x^2 \left(\frac{1-f}{n'} \right)$$

$$E(e_0 e'_1) = \frac{1}{\bar{Y}\bar{X}} \frac{(1-f)}{n'} S_{yx}, \quad E(e_0 e'_2) = \frac{1}{\bar{Y}\bar{Z}} \frac{(1-f)}{n'} S_{yz}$$

$$E(e_1 e'_2) = \frac{1}{\bar{X}\bar{Z}} \left(\frac{1-f}{n'} \right) S_{xz}, \quad E(e'_1 e'_2) = \frac{1}{\bar{X}\bar{Z}} \left(\frac{1-f}{n'} \right) S_{xz}$$

where,

$$f = \frac{n'}{M}, \quad S_x^2 = \frac{1}{M-1} \sum_{k=1}^N \sum_{h=1}^{M_k} (x_{kh} - \bar{X}_k)^2$$

$$S_y^2 = \frac{1}{M-1} \sum_{k=1}^N \sum_{h=1}^{M_k} (y_{kh} - \bar{Y}_k)^2$$

$$S_z^2 = \frac{1}{M-1} \sum_{k=1}^N \sum_{h=1}^{M_k} (z_{kh} - \bar{Z}_k)^2$$

$$S_{xk}^2 = \frac{1}{M_k-1} \sum_{h=1}^{M_k} (x_{kh} - \bar{X}_k)^2$$

$$S_{yk}^2 = \frac{1}{M_k-1} \sum_{h=1}^{M_k} (y_{kh} - \bar{Y}_k)^2$$

$$S_{zk}^2 = \frac{1}{M_k-1} \sum_{h=1}^{M_k} (z_{kh} - \bar{Z}_k)^2$$

$$S_{yx} = \frac{1}{M-1} \sum_{k=1}^N \sum_{h=1}^{M_k} (y_{kh} - \bar{Y}_k)(x_{kh} - \bar{X}_k)$$

$$S_{yz} = \frac{1}{M-1} \sum_{k=1}^N \sum_{h=1}^{M_k} (y_{kh} - \bar{Y}_k)(z_{kh} - \bar{Z}_k)$$

$$S_{xz} = \frac{1}{M-1} \sum_{k=1}^N \sum_{h=1}^{M_k} (x_{kh} - \bar{X}_k)(z_{kh} - \bar{Z}_k)$$

By putting the above values in (5), $\hat{Y}_{PPS}^{ds}$ have expressed as

$$\hat{Y}_{PPS}^{ds} - \bar{Y} = \bar{Y} \left[\begin{aligned} & \frac{1}{2} (2e_0 - e'_1 - e'_2 + e_1) + (\alpha_2 - \alpha_1) e'_2 - e_1 + e'_1 + \\ & \frac{1}{8} \left(3e_2^2 - e_1^2 - e_1'^2 + 4e_0 e_1 - 4e_0 e'_1 - 4e_0 e'_2 - \right. \\ & \quad \left. 2e_1 e'_1 - 2e_1 e'_2 + 2e'_1 e'_2 \right) + \\ & \left(\frac{1}{2} ((\alpha_2 - \alpha_1 - 1) + (\alpha_2 - \alpha_1)^2) \right. \\ & \quad \left. \left(e_2'^2 - (\alpha_2 - \alpha_1) e_1 e'_2 - \frac{1}{2} e_1'^2 - (\alpha_2 - \alpha_1) e_1 e'_2 \right) \right. \\ & \quad \left. + (\alpha_2 - \alpha_1) e_0 e'_2 - e_0 e_1 + e_0 e'_1 + e_1^2 \right) \end{aligned} \right] \quad (6)$$

Finally, upto fda $B(\hat{Y}_{RRS}^{ds})$ is obtained as

$$B(\hat{Y}_{PPS}^{ds}) = \left[\frac{1}{n'} \sum_{k=1}^N W_k \left(1 - \frac{1}{u_k} \right) \frac{1}{\bar{X}} \left(\frac{1}{2} S_{yxk} + \frac{5}{8} R_1 S_{xk}^2 \right) + \left(\frac{1-f}{n'} \right) \left\{ \left(\frac{\alpha_2 - \alpha_1}{\bar{Z}} \right) \left(S_{yz} - \frac{1}{2} R_2 S_z^2 \right) + \left(\frac{5}{8} R_2 S_z^2 - \frac{1}{2} S_{yz} \right) \right\} \right] \quad (7)$$

Square and expectation of (6) provides $MSE \hat{Y}_{PPS}^{ds}$ as

$$E[\hat{Y}_{PPS}^{ds} - \bar{Y}]^2 = \hat{Y}^2 E \left[\frac{2e_0 - e'_1 - e'_2 + e_1}{2} + (\alpha_2 - \alpha_1)e'_2 - e_1 + e'_1 \right]^2$$

$$MSE(\hat{Y}_{PPS}^{ds}) = \frac{1}{4} \bar{Y}^2 E[4e_0^2 + e_1'^2 + ((\alpha_2 - \alpha_1)(4\alpha_2 - 4\alpha_1 - 1) + 1)e_2'^2 + e_1'^2 + 4e_0e'_1 + (4(2\alpha_2 - 2\alpha_1 - 1))e_0e'_2 - 4e_0e_1 - (2(2\alpha_2 - 2\alpha_1 - 1))e'_1e'_2 - 6e_1e'_1 - (2(2\alpha_2 - 2\alpha_1 - 1))e_0e'_2]$$

$$MSE(\hat{Y}_{PPS}^{ds}) = \frac{\bar{Y}^2}{4} \left[\frac{4}{\bar{Y}^2} \left\{ \frac{1}{n'} \sum_{k=1}^N W_k \left(\frac{1}{u_k} - 1 \right) S_{yk}^2 \right\} + ((\alpha_2 - \alpha_1)(4\alpha_2 - 4\alpha_1 - 1) + 1) \frac{1}{\bar{Z}^2} S_z^2 \left(\frac{1-f}{n'} \right) + \frac{8}{\bar{X}^2} S_x^2 \left(\frac{1-f}{n'} \right) + \frac{(4(2\alpha_2 - 2\alpha_1 - 1))}{\bar{Y}\bar{Z}} \left(\frac{1-f}{n'} \right) S_{yz} - \frac{8(\alpha_2 - \alpha_1)}{\bar{X}\bar{Z}} \left(\frac{1-f}{n'} \right) S_{xz} - \frac{4}{\bar{Y}\bar{X}} \left\{ \frac{1}{n'} \sum_{k=1}^N W_k \left(\frac{1}{u_k} - 1 \right) S_{yxk} \right\} + \frac{4}{\bar{X}\bar{Y}} \left(\frac{1-f}{n'} \right) S_{xy} \right]$$

finally, fda, MSE of $\hat{Y}_{PPS}^{ds}$ is obtained as

$$MSE(\hat{Y}_{PPS}^{ds}) = \left[\left(\frac{1-f}{n'} \right) \left(\frac{1}{4} ((\alpha_2 - \alpha_1)(4\alpha_2 - 4\alpha_1 - 1) + 1) R_2^2 S_z^2 + 2R_1^2 S_x^2 + 4(2\alpha_2 - 2\alpha_1 - 1) R_2 S_{yz} + S_y^2 + 8(\alpha_2 - \alpha_1) R_1 R_2 S_{xz} \right) + \frac{1}{n'} \sum_{k=1}^N W_k \left(\frac{1}{u_k} - 1 \right) (S_{yk}^2 - R_1 S_{yxk}) \right] \quad (8)$$

3. Comparisons of Estimators

Here, the suggested novel estimator is being compared with previously exist estimators w.r.t their efficiencies.

In DSS, the $var(\bar{y}_{ds})$, MSE of author and Ige & Tripathi estimator are respectively given by

$$V(\bar{y}_{ds}) = S_y^2 \left(\frac{1-f}{n'} \right) + \frac{1}{n'} \sum_{k=1}^N W_k S_{yk}^2 \left(\frac{1}{u_k} - 1 \right) \quad (9)$$

$$MSE(\hat{Y}_{PP}^{ds}) = S_y^2 \left(\frac{1-f}{n'} \right) + \frac{1}{n'} \sum_{k=1}^N W_k \left(\frac{1}{u_k} - 1 \right) \times (S_{yk}^2 + R_1^2 S_{xk}^2 - 2R_1 S_{yxk}) \quad (10)$$

$$MSE(\hat{Y}_{PPE}^{ds}) = S_y^2 \left(\frac{1-f}{n'} \right) + \frac{1}{n'} \sum_{k=1}^N W_k \left(\frac{1}{u_k} - 1 \right) \times \left[S_{yk}^2 + \frac{R_1^2}{4} \left(S_{xk}^2 - \frac{S_{yxk}}{R_1} \right) \right] \quad (11)$$

By comparing (8), (9), (10) and (11) one can easily found that the suggested novel estimator named as $\hat{Y}_{PPS}^{ds}$ is better w.r.t. efficiency comparison then (i) $\bar{y}_{ds}$ if

$$\left(\frac{1-f}{n'} \right) \left(\frac{1}{4} ((\alpha_2 - \alpha_1)(4\alpha_2 - 4\alpha_1 - 1) + 1) R_2^2 S_z^2 + 2R_1^2 S_x^2 + 4(2\alpha_2 - 2\alpha_1 - 1) R_2 S_{yz} + 8(\alpha_2 - \alpha_1) R_1 R_2 S_{xz} \right) < \sum_{k=1}^N W_k \left(\frac{1}{u_k} - 1 \right) (R_1 S_{yxk}) \quad (12)$$

(ii) Ige-Tripathi estimator $\hat{Y}_{PP}^{ds}$ if

$$\left(\frac{1-f}{n'} \right) \left(\frac{1}{4} ((\alpha_2 - \alpha_1)(4\alpha_2 - 4\alpha_1 - 1) + 1) R_2^2 S_z^2 + 2R_1^2 S_x^2 + 4(2\alpha_2 - 2\alpha_1 - 1) R_2 S_{yz} + 8(\alpha_2 - \alpha_1) R_1 R_2 S_{xz} \right) < \sum_{k=1}^N W_k \left(\frac{1}{u_k} - 1 \right) (R_1 S_{yxk} - R_1^2 S_{xk}^2) \quad (13)$$

(iii) Estimator $\hat{Y}_{PPE}^{ds}$ if

$$\left(\frac{1-f}{n'} \right) \left(\frac{1}{4} ((\alpha_2 - \alpha_1)(4\alpha_2 - 4\alpha_1 - 1) + 1) R_2^2 S_z^2 + 2R_1^2 S_x^2 + 4(2\alpha_2 - 2\alpha_1 - 1) R_2 S_{yz} + 8(\alpha_2 - \alpha_1) R_1 R_2 S_{xz} \right) < \sum_{k=1}^N W_k \left(1 - \frac{1}{u_k} \right) \left(\frac{1}{4} R_1^2 S_{xk}^2 + \frac{5}{4} R_1 S_{yxk} \right) \quad (14)$$

4. Empirical Study

Here, one natural data of population is considered for the purpose to check performance of the suggested estimator by comparing it with considered estimators with the help of case study with different value of α_1 and α_2 .

4.1. Population I [Source: Murthy (1967), p.228]

Y : As Outcome,

X : As Fixed capital and

Z : As No. of workers,

Table 1: Parameters for Stratified Sampling Analysis

Parameter	Strata (I)	Strata (II)	Parameter	Strata (I)	Strata (II)
N_h	5	5	S_{yzh}	15.85	56.05
n_h	2	2	S_{xzh}	60.45	10.85
n'_h	4	4	R_h	0.1266	0.0216
$\bar{Y}_h$	3394.6	3590.2	S_y^2	5749.9	
$\bar{X}_h$	410.2	474.4	S_z^2	22.98	
$\bar{Z}_h$	71.2	80	S_x^2	637.77	
S_{yh}	68.79	90.582	S_{yz}	31.968	
S_{xh}	26.864	26.708	S_{yx}	1023.2	
S_{zh}	2.3875	6.782	S_{xz}	31.689	
S_{yxh}	49.85	2252.41			

Table 2: PREs of $\bar{y}_{ds}$, $\hat{\bar{y}}_{pp}^{ds}$, $\hat{\bar{y}}_{ppe}^{ds}$ and $\hat{\bar{y}}_{pps}^{ds}$ with respect to $\bar{y}_{ds}$

Estimators	PRE's		
	$(u_1, u_2, \alpha_1, \alpha_2)=(0.5, 0.5, 3, 4)$	$(u_1, u_2, \alpha_1, \alpha_2)=(0.6, 0.5, 2, 3)$	$(u_1, u_2, \alpha_1, \alpha_2)=(0.6, 0.55, 4, 5)$
$\bar{y}_{ds}$	100.00	100.00	100.00
$\hat{\bar{y}}_{pp}^{ds}$	102.59	102.2978	102.461
$\hat{\bar{y}}_{ppe}^{ds}$	58	93.6244	86.5857
$\hat{\bar{y}}_{pps}^{ds}$	108.3275	123.1179	114.184

Table 3: Numerical values of MSE given in (12), (13) and (14)

Estimators	Population		
	$(u_1, u_2, \alpha_1, \alpha_2)=(0.5, 0.5, 3, 4)$	$(u_1, u_2, \alpha_1, \alpha_2)=(0.2, 0.4, 2, 3)$	$(u_1, u_2, \alpha_1, \alpha_2)=(0.4, 0.6, 4, 5)$
$MSE(\hat{\bar{y}}_{pps}^{ds}) < MSE(\bar{y}_{ds})$	1277.2077 < 1383.56	1043.7134 < 1284.9826	1043.7012 < 1191.74188
$MSE(\hat{\bar{y}}_{pps}^{ds}) < MSE(\bar{y}_{pp}^{ds})$	1277.2077 < 1348.57	1043.7134 < 1250.0089	1043.7012 < 1163.119
$MSE(\hat{\bar{y}}_{pps}^{ds}) < MSE(\hat{\bar{y}}_{ppe}^{ds})$	1277.2077 < 2346.225	1043.7134 < 1372.4869	1043.7012 < 1376.3719

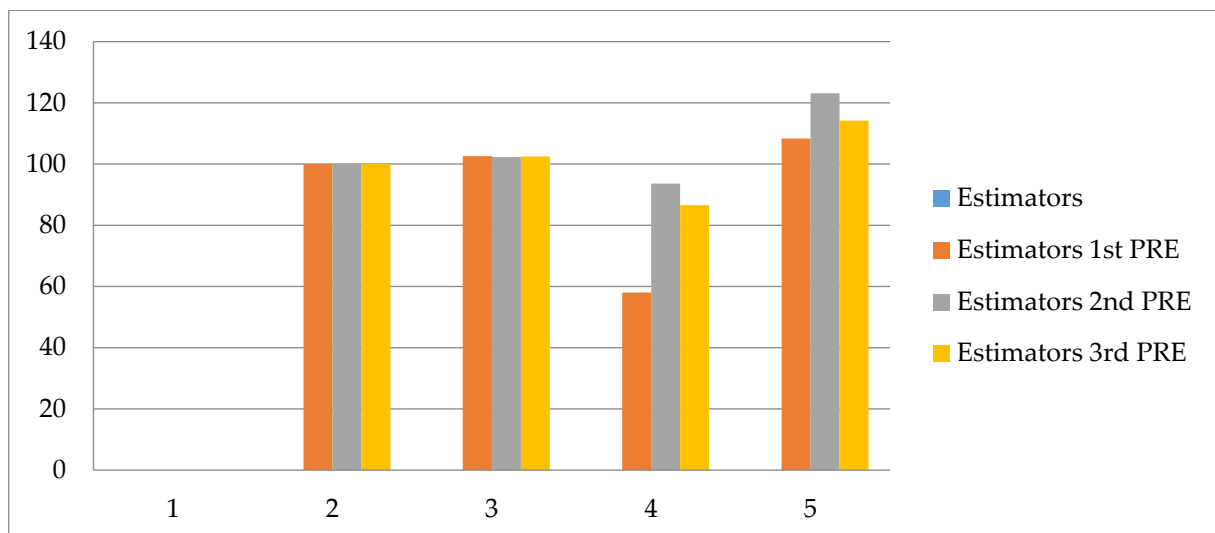


Figure 1: Graphical representations of PRE

5. Conclusions

In this article, the product in product typed estimator with exponential and log function for the population-

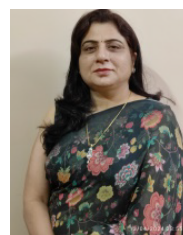
mean is developed. Since ratio typed estimator outperforms the simple mean estimator, we have substituted the sample mean derived from 1st phase sample with a ratio estimator that makes the use of the

known $\bar{Z}$. Section (4) presents empirical examples that support the previously stated notion that was used to build $\hat{Y}_{PPS}^{ds}$. Table (2) shows that out of all the estimators that were investigated, estimator $\hat{Y}_{PPS}^{ds}$ had the highest percent relative efficiency. The conditions under which $\hat{Y}_{PPS}^{ds}$ have a lower MSE are shown in Table (3). The best per cent relative efficiency of $\hat{Y}_{PPS}^{ds}$ is shown in Table (2) and figure 1 represents the graphical explanations for each estimator and shows the results are best for suggested estimators. All the requirements acquired in Section (3) are met, as shown in Table (3). Thus, if the conditions which are given in Section (3) are satisfied, $\hat{Y}_{PPS}^{ds}$ is recommended for practical use because of its higher percent relative efficiency comparatively in the field as an alternative to the $\bar{y}_{ds}$ (the usual unbiased estimator), $\hat{Y}_{PP}^{ds}$ (Ige & Tripathi ratio estimator) and $\hat{Y}_{PPE}$ (ratio type exponential estimator) given by Tailor in the case of the DSS technique.

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